ACT® International Subject Tests

Mathematics 2 Reference Sheet

Lines

Standard Form	$Ax + By = C$	A, B, and C are constants, where $A \neq 0$ or $B \neq 0$.
Slope-Intercept Form	$y = mx + b$	m = slope
Point-Slope Form	$y - y_1 = m(x - x_1)$	b = y-intercept
Slope	$m = \frac{y_2 - y_1}{x_2 - x_1}$	(x_1,y_1) and (x_2,y_2) are 2 points.

Quadratics

General Form	$ax^2 + bx + c = 0$	a, b, and c are constants, where $a \neq 0$.
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	

Coordinate Geometry

Midpoint	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	(x_1,y_1) and (x_2,y_2) are 2 points.
Distance	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	

Area, Volume, and Surface Area of Polygons and Solids

Triangle	$A = \frac{1}{2}bh$	A = area
Parallelogram	$A = bh$	b = base
Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$	h = height
Regular Polygon	$A = \frac{1}{2}ap$	a = apothem
Prism	$V = Bh$	p = perimeter
Right Prism	$SA = 2B + Ph$	V = volume
Circular Cylinder	$V = \pi r^2 h$	B = area of base
Right Circular Cylinder	$SA = 2\pi r^2 + 2\pi rh$	SA = surface area
Pyramid	$V = \frac{1}{3}Bh$	P = perimeter of base
Right Pyramid	$SA = B + \frac{1}{2}Ps$	r = radius
Circular Cone	$V = \frac{1}{3}\pi r^2 h$	s = slant height
Right Circular Cone	$SA = \pi r^2 + \pi rs$	$\pi \approx 3.14$
Sphere	$V = \frac{4}{3}\pi r^3$	
	$SA = 4\pi r^2$	

Circles

Center-Radius Form	$(x - h)^2 + (y - k)^2 = r^2$	center (h,k)
Area	$A = \pi r^2$	r = radius
Circumference	$C = \pi d = 2\pi r$	A = area
Area of Sector	$A = \frac{\theta}{360}\pi r^2$	C = circumference
		d = diameter
		θ = degree measure of central angle
		$\pi \approx 3.14$

Parabolas

Opening vertically	$y = a(x - h)^2 + k$	a = constant
axis of symmetry	$x = h$	(h,k) = vertex
focus	$\left(h, k + \frac{1}{4a}\right)$	
directrix	$x = k - \frac{1}{4a}$	
Opening horizontally	$x = a(y - k)^2 + h$	
axis of symmetry	$y = k$	
focus	$\left(h + \frac{1}{4a}, k\right)$	
directrix	$y = h - \frac{1}{4a}$	

Ellipses

Major Axis Horizontal	$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$	a, b = positive constants where $a > b$
foci	$(h \pm c, k)$	$c = \sqrt{a^2 - b^2}$
Major Axis Vertical	$\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1$	A = area
foci	$(h, k \pm c)$	$\pi \approx 3.14$
Area	$A = \pi ab$	

Hyperbolas

Transverse Axis Horizontal	$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$	a, b = positive constants where $a > b$
foci	$(h \pm c, k)$	$c = \sqrt{a^2 + b^2}$
Transverse Axis Vertical	$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$	
foci	foci $(h, k \pm c)$	

Triangles

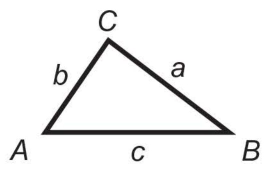
Law of Sines $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of Cosines $c^2 = a^2 + b^2 - 2ab\cos C$

Area of a Triangle $\text{Area} = \frac{1}{2}bc\sin A$

$= \sqrt{s(s-a)(s-b)(s-c)}$

Pythagorean Theorem $a^2 + b^2 = c^2$, if $\angle C$ is a right angle



s = semiperimeter = $\frac{(a+b+c)}{2}$

Sequences and Series

Arithmetic Sequence	$a_n = a_1 + (n-1)d$	a_n = nth term
Arithmetic Series	$s_n = \frac{n}{2}(a_1 + a_n)$	n = term number
Geometric Sequence	$a_n = a_1(r^{n-1})$	d = common difference
Finite Geometric Series	$s_n = \frac{a_1 - a_1 r^n}{1-r}$ where $r \neq 1$	r = common ratio
Infinite Geometric Series	$s = \frac{a_1}{1-r}$ where $\lvert r \rvert < 1$	s_n = sum of the first n terms
Combinations	${}_kC_m = C(k,m) = \frac{k!}{(k-m)!\,m!}$	s = sum of all the terms
Permutations	${}_kP_m = P(k,m) = \frac{k!}{(k-m)!}$	k = number of objects in the set
		m = number of objects selected

Interest

Simple Interest	$I = prt$	I = interest
Compound Interest	$A = p\left(1 + \frac{r}{n}\right)^{nt}$	p = principal
Continuously Compounded Interest	$A = pe^{rt}$	r = annual interest rate
		t = number of years
		A = amount of money after t years
		n = compound periods per year
		$e \approx 2.718$

Exponential Growth and Decay

Periodic	$N_t = N_0(1 + r)^t$	N_t = value at time t or after t time periods
Continuous	$N_t = N_0 e^{rt}$	r = rate of growth
		t = time or number of time periods
		$e \approx 2.718$

Polar Coordinates and Vectors

De Moivre's Theorem

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

r = radius, distance from origin

θ = angle measure in standard position

n = exponent

Conversion: Polar to Rectangular Coordinates

$$x = r\cos\theta$$
$$y = r\sin\theta$$

Conversion: Rectangular to Polar Coordinates

$$r = \sqrt{x^2 + y^2},\ \theta = \arctan\frac{y}{x},\ \text{when } x > 0$$
$$\theta = \pi + \arctan\frac{y}{x},\ \text{when } x < 0$$

Product of Complex Numbers in Polar Form

$$[r_1(\cos\theta_1 + i\sin\theta_1)][r_2(\cos\theta_2 + i\sin\theta_2)] = r_1r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$$

Inner Product of Vectors

$$\mathbf{a} \bullet \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$$

$\mathbf{a} = \langle a_1, a_2\rangle$ vector in the plane

$\mathbf{a} = \langle a_1, a_2, a_3\rangle$ vector in space

Matrices

Determinant of a 2 × 2 Matrix

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

Determinant of a 3 × 3 Matrix

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix} = a \cdot \det\begin{bmatrix} e & f \\ h & j \end{bmatrix} - b \cdot \det\begin{bmatrix} d & f \\ g & j \end{bmatrix} + c \cdot \det\begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

Inverse of a 2 × 2 Matrix

$$M^{-1} = \frac{1}{\det M}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ where } M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Trigonometry

Sum and Difference Identities

α, β, θ = angle measures in standard position

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

Double-Angle Identities

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Half-Angle Identities

$$\sin \frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\tan \frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}, \text{ where } \cos \alpha \neq -1$$

Miscellaneous

Distance, Rate, Time $D = rt$

Direct Variation (y varies directly with x) $y = kx$

Indirect Variation (y varies indirectly with x) $y = \frac{k}{x}$

D = distance

r = rate

t = time

k = variation constant