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Bayes, Hume, and Miracles

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Recent attempts to cast Hume's argument against miracles in a Bayesian form are examined. It is shown how the Bayesian apparatus does serve to clarify the structure and substance of Hume's argument. But the apparatus does not underwrite Hume's various claims, such as that no testimony serves the establish the credibility of a miracle; indeed, the Bayesian analysis reveals various conditions under which it would be reasonable to reject the more interesting of Hume's claims.

Recent articles by Dawid and Gillies (1989), Gillies (1991), Owen (1987), and Sobel (1987, 1991) have applied the machinery of modern Bayesianism to Hume's argument against miracles. There are some historical grounds for a Bayes-Hume connection, albeit of a somewhat tenuous kind. The current dating of Bayes' essay supports the conjecture that Bayes had read and was in part reacting to Hume's skeptical attack on induction.[1] In the other direction, Richard Price, who arranged for the posthumous publication of Bayes' essay, produced a work entitled *Four Dissertations*, the fourth of which cited Bayes' essay as part of an attack on Hume's argument.[2] Hume acknowledged Price's work in a cordial letter,[3] and Price returned the compliment by praising Hume in the second edition of *Four Dissertations* as "a writer whose genius and abilities are so distinguished, as to be above any of *my* commendations" (1768, p. 382). Nevertheless, the application to Hume of what we now call Bayesianism is anachronistic. For example, Bayes' theorem, so-called, is not to be found in either Price's work or in Bayes' original essay.[4]

Despite its anachronistic character, I agree with Dawid, Gillies, et al. that modern Bayesianism does serve to clarify the structure and substance of Hume's argument. I differ with the authors of these excellent articles on details of the interpretation of Hume and, more importantly, on what the analysis shows about the force of Hume's argument vs. what it shows about Bayesianism. While I am under no illusion that I can bring closure to this complex and endlessly fascinating topic, I hope to identify more accurately the points which a resolution of the problems involved must address.

1. *Hume's Maxim on Establishing a Miracle*

In Section X ("Of Miracles") of the *Enquiry Concerning Human Understanding*[5] Hume offered the following Maxim:

> That no testimony is sufficient to establish a miracle, unless the testimony be of such a kind that its falsehood would be more miraculous, than the fact, which it endeavors to establish; and even in that case there is a mutual destruction of arguments, and the superior only gives us an assurance suitable to that degree of force, which remains, after deducting the inferior. (pp. 115-116)

In offering a Bayesian explication of the first part of Hume's Maxim, I will adopt Dawid and Gillies (1989) notation, where A is the proposition asserting the occurrence of the miraculous event in question, a is the proposition that a witness W has testified to the occurrence of the event, and K is the background knowledge. Since we are in a context where we know that W has testified, we should according to one standard Bayesian line conditionalize on a.[6] So the relevant probability of the falsehood of the testimony is the conditional probability $\Pr(\neg A/a\&K)$, and the relevant probability of the fact which the testimony endeavors to establish is the conditional probability $\Pr(A/a\&K)$. And since for one thing to be more miraculous than another is for the former to be less probable than the latter, the unless clause of Hume's Maxim is rendered

$$\Pr(A/a\&K) > \Pr(\neg A/a\&K). \qquad (1)$$

On this reading the first part of Hume's Maxim is surely correct. For the testimony to establish the credibility of a miracle in the sense of making the miracle probable, a must combine with K so that

$$\Pr(A/a\&K) > .5, \qquad (1')$$

and (1) is necessary and sufficient for (1′). It could be objected to my reading that it makes the first part of Hume's Maxim a platitude of Bayes-speak. On the contrary, I take this consequence to be a virtue especially since Hume says that he is offering a "general maxim."

Gillies (1991) and Sobel (1991) propose to interpret the unless clause of Hume's Maxim as asserting that

$$\Pr(A/K) > \Pr(a\&\neg A/K). \qquad (2)$$

It has to be admitted that (2) fits better than (1) with Hume's declaration that the argumentation of Part I of his essay is based on the assumption that the "falsehood of the testimony [to the miracle] would be a real prodigy." (p. 116); for if the prodigy consists of a low value for $\Pr(\neg A/a\&K)$ rather than for $\Pr(a\&\neg A/K)$, then (1) and (1′) automatically hold. But perhaps Hume's declaration was only meant to signal that in Part I he was not addressing the credibility problems that attach to testimonial evidence for miracles deemed to have religious significance. Moreover, immediately after stating the Maxim, Hume gives the following illustration:

> When anyone tells me, that he saw a dead man restored to life, I immediately consider with myself, whether it be more probable, that this person should

> either deceive or be deceived, or that the fact, which he relates, should really have happened. I weigh the one miracle against the other; and according to the superiority, which I discover, I pronounce my decision, and always reject the greater miracle. (p. 116)

The most straightforward reading of the probabilities involved is in terms of the conditional probabilities in (1).

Of course, if K contains a then (2) coincides with (1) (since $\Pr(a\&\neg A/a\&X) = \Pr(\neg A/a\&X)$).[7] But if not, then (2) does not coincide with (1); in particular, (2) is not sufficient for (1) or (1′), which is awkward since Hume sometimes talks as if the fulfilment of the unless clause of his Maxim is sufficient to establish a miracle, as when he writes that "If the falsehood of his testimony would be more miraculous, than the event which he relates; then, and not till then, can he pretend to command my belief or opinion" (p. 116).[8]

In any case, Gillies and Sobel are correct in noting that, assuming $\Pr(a/K) > 0$, (2) is a necessary condition for (1). Applying Bayes' theorem to both sides of (1) and performing a couple of elementary operations leads to

$$\Pr(A/K) > \frac{\Pr(a\&\neg A/K).}{\Pr(a/A\&K)} \tag{3}$$

Since $\Pr(a/A\&K) \leq 1$, (3) can hold only if (2) holds.

Since Hume himself did not use probability notation and was evidently not familiar with the probability calculus, it is hardly surprising that his Maxim is ambiguous as viewed through the lens of probability. Various ambiguities were pointed out as early as 1838 by Charles Babbage in his *Ninth Bridgewater Treatise*. One of Babbage's readings is in line with (1), my preferred interpretation. Babbage also suggests reading the unless clause of the Maxim as requiring that $\Pr(A/K) > \Pr(a/\neg A\&K)$, which is less useful since it is neither necessary nor sufficient for (1).[9]

Whatever Hume's intentions, it is (1) that counts for deciding whether the testimony has established the credibility of a miracle. Thus, in what follows I will use (1) as the Bayesian reading of the unless clause of Hume's Maxim. A little further manipulation of the equivalent (3) of (1) produces another and, for Hume's purposes, more useful necessary and sufficient condition for (1):

$$\Pr(A/K) > \Pr(a/\neg A\&K) \mathrm{x} [(1 - \Pr(A/K))/\Pr(a/A\&K)]. \tag{4}$$

I will have occasion below to use (4) in assessing the strength of Hume's argument.

2. *Hume's Goal: Strong Form*

Any assessment of Hume's argument against miracles must, of course, start from the goals he was trying to reach. As a first rough cut we can say that

Hume had two goals, corresponding to his division of "Of Miracles" into two parts. Part I purports to supply a "*proof* ... against the existence of any miracle" (p. 115), and this is so even on the assumption that the falsehood of the testimony, upon which the miracle is to be founded, "would be a real prodigy" (p. 116). Part II provides, as it were, a fall back position, by arguing for a pair of more modest claims. The first is that the assumption in question is unwarranted in all actual historical cases, with the upshot that no testimony has ever established the probability of a miracle. The second claim is that when we take into account the special features attending alleged religious miracles, the assumption in question always fails, with the upshot that no testimony can ever have the force to establish a miracle as "a just foundation of religion" (p. 127). In fact, however, the division is no so neat. Part II repeats some of the sentiments of Part I. And in the editions of the *Enquiry* prior to 1768, Part II contained the assertion that "... it appears that no testimony for any kind of miracle can ever possibly amount to a probability, much less a proof."[10] I will not attempt to resolve this and other puzzles of organization to be remarked on below, but instead I turn to a critical examination of Hume's claims.

When Hume claims in Part I to offer a "*proof* ... against the existence of any miracle," I take him at his word: he means his argument to apply to any miracle, not just miracles that are supposed to have religious significance. This literal reading seems to be belied by a footnote. After first defining a miracle as "a violation of the laws of nature" (p. 114), Hume adds the qualification that "A miracle may be accurately defined, *a transgression of a law of nature by a particular volition of the Deity, or by the interposition of some invisible agent*" (p. 115, n. 1). I regard this note as an organizational aberration, but whether or not I am correct, the key point here is that nothing in Hume's argument for his strong claim in Part I rests on a presumed supernatural cause of the violation of the law. Hume's note is, however, relevant to a kind of last ditch position which will be examined below in section 4.

Hume's "proof" is admirably brief. The challenge here is to find a plausible Bayesian reading of it. "A miracle," according to Hume "is a violation of a law of nature; and as a firm and unalterable experience has established these laws, the proof against miracles, from the very nature of the fact, is as entire as any argument from experience can possibly be imagined" (p. 114). A little later Hume argues that because the miraculous type event has never been observed in any age or country

> There must, therefore, be a uniform experience against every miraculous event, otherwise the event would not merit that appellation. And as uniform experience amounts to a proof, there is here a direct and *full* proof, from the nature of the fact, against the existence of any miracle ... (p. 115)

Perhaps in Bayesian terms this should be taken to mean that if A is a counterinstance to a well confirmed (putative) law of nature L, and if K summarizes "firm and unalterable experience," then $Pr(A/K) = 0$ and, consequently $Pr(A/a\&K) = 0$, which in turn means that (1) fails.[11] If L can be thought of as an infinite conjunction of a countable number of instances in the sense that $Pr(L) = \lim_{n \to \infty} Pr(\neg A(1)\&\neg A(2)\&...\&\neg A(n))$, then setting $Pr(A(i)/K) = 0$ for each i = 1, 2, ... entails that $Pr(L/K) = 1$, which grates against both common sense and actual scientific practice. Scientists not uncommonly spend many hours and many dollars searching for events of a type that past experience tells us never have occurred (e.g. proton decay). Such practice is hard to understand if the probability of such an event is flatly zero and the probability of the putative law asserting the non-occurrence of this type of event is unity. Richard Price (1768) argued as much.

> It must, however, be remembered, that the greatest uniformity and frequency of experience will not afford a proper *proof*, that an event will happen in a future trial, or even render it so much as probable, that it will *always* happen in all future trials... [L]et us suppose a solid which, for ought we know, may be constituted in any one of an infinity of different ways, and that we can judge of it only from experiments made in throwing it ... But though we knew, that it had turned the same face in every trial a million of times, there would be no *certainty* that it would turn this face again in any particular future trial, nor even the least *probability*, that it would *never* turn any other face (pp. 392-393). These observations are applicable, in the exactest manner, to what passes in the course of nature, as far as *experience* is our guide. Upon observing, that any natural event has happened often or invariably, we have only reason to expect that it will happen again, with an assurance proportioned to the frequency of our observations. But, we have no *absolute proof* that it will happen again in any particular future trial; nor the least reason to believe that it will always happen. (p. 395).

It is ironic indeed that Hume's strong claim against miracles makes him out to be much less of an inductive skeptic than his opponent Price.[12]

Hume also held that "the evidence, resulting from the testimony, admits of diminution, greater or less in proportion as the fact is more or less unusual" (p. 113).[13] He might then have reasoned that since a miraculous event is not merely an unusual one but an extremely unlikely one, the testimony is so diminished that it cannot possibly establish a miracle in the sense of (1). If so, he was mistaken. Take the strength of the evidence resulting from the testimony to be measured by $Pr(A/a\&K)$. Then Bayes' theorem shows that Hume was correct to the extent that this strength is directly proportional to $Pr(A/K)$. So if the unusualness of the event is reflected in the assignment of a low prior probability, then the unusualness of the event does, other things equal, diminish the strength of the testimony. But other things need not be equal. And Bayes' theorem shows how and under what circumstances testimonial evidence can make A more probable than not as long as we set—as I argued above we should—$Pr(A/K) > 0$.

There is a sense in which we did not need to go through this exercise of first searching the text of "Of Miracles" to find possible motivations for Hume's strong claim against miracles, and then rejecting each in turn; for one knows in advance that this claim cannot stand. Every student of "Of Miracles" knows the dilemma that faces Hume's definition of miracles as a violation of laws of nature. In the Bayesianized setting, this dilemma takes the following form. If a law of nature is defined as true general proposition and, therefore, one without exceptions, then there cannot—logically cannot—be a miracle, and no Bayesianizing is required to show it. On the other hand, if by a law Hume meant a putative law—that is, a proposition lawlike in form (here fill in your favorite account of lawlikeness), which has no known counterinstances and many known positive instances—then we certainly do not want to be committed to the position that no amount of testimonial evidence can ever make us reasonably sure (in the sense of a posterior probability greater than .5) that the proposition fails. If Bayesianism entailed such a position—and no extant form of it does—then Bayesianism would be suspect.

How could Hume have gone so wrong? The answer lies in Hume's crude view about how probability considerations are to be applied. Roughly, his idea was that when we are dealing with a type of event which, in the appropriate circumstances, sometimes occurs and sometimes not, then the probability calculus is to be brought into play in forming estimates of the chances that the event will occur in some future trial; but if the event has invariably occurred in the appropriate circumstances, then on Hume's view probability considerations are irrelevant since we have a full "proof."[14]

3. *Hume's Goal: Modest Forms*

In Part II Hume reviews a number of historical cases and concludes—in editions after 1768—that "Upon the whole, then, it appears that no testimony for any kind of miracle ever has amounted to a probability, much less a proof" (p. 127). Later in the same paragraph Hume makes a stronger and more interesting in-principle claim that "no human testimony can have such a force as to prove a miracle, and make it a just foundation for any such [i.e. any popular] religion." On one plausible reading this latter claim is to be understood as the assertion that no testimony can establish the violation of a (putative) law when the violation is deemed to have religious significance. The type of example Hume has in mind here is a resurrection, a walking on water, and the like. In the following section I will consider a weaker reading of the in-principle claim.

Hume's argument for the in-principle claim is based on the notion that "if the spirit of religion join itself to the love of wonder, there is an end of common sense; all human testimony, in these circumstances, loses all pretensions to authority" (p. 117). The argument offered refers back to the second half of the Maxim, which suggests a kind of subtraction procedure. The substraction procedure becomes more explicit in Part II:

> It is experience only, which gives authority to human testimony; and it is the same experience, which assures us of the laws of nature. When, therefore, these two kinds of experience are contrary, we have nothing to do but subtract the one from the other, and embrace the opinion, either on one side or the other, with that assurance which arises from the remainder. But according to the principle here explained, this subtraction, with regard to all popular religions, amounts to an entire annihilation; and therefore we may establish it as a maxim, that no human testimony can have such force as to prove a miracle, and make it a just foundation for any such system of religion. (p. 127)

This subtraction procedure may seem to involve an illicit double counting. If $\Pr(A/a\&K) > .5$, then that's the way it is; the dangers of self-deception and deceit in cases where the alleged miracle is deemed to have religious significance have already been taken into account. But there is a more plausible version of Hume's argument that can be rendered by using formula (4). From the definition of miracle we can agree that the left hand side of (4) has a tiny but non-zero value. For sake of illustration let us set $\Pr(A/K) = 10^{-8}$. The witnesses who testify to miraculous events deemed to have religious significance tend to be religious believers or those predisposed to religious belief. It is then fair to assume that such a witness will almost surely report a miraculous event of alleged religious significance if she observes it. Thus, $\Pr(a/A\&K)$ is very near 1, though perhaps not as near as $1 - \Pr(A/K) = 1 - 10^{-8}$, with the upshot that the square bracketed term on the right hand side of (4) has a value of 1 or somewhat greater than 1. So for (4), and consequently for (1), to obtain, it is necessary that $\Pr(A/K) > \Pr(a/\neg A\&K)$. But surely (the Bayesianized Hume may argue) the probability that the witness will offer a testimonial on an occasion when the miraculous event does not occur far exceeds 10^{-8}. For witnesses who are already religious believers or who are predisposed to religious belief are vulnerable to self-deception and to the deception of others, and the converted are not above using deceit to win over the unconverted.

It is worth emphasizing that this version of Hume's argument does not rest on the contentious principle that the evidence of testimony is diminished in direct proportion to the improbability of the event testified to. Price countered that "improbabilities *as such* do not lessen the capacity of testimony to report the truth" (1768, p. 413). As an example, he noted that our inclination to believe a newspaper report that, say, ticket #11,423 was drawn in the lottery is not diminished in proportion as the number of lottery tickets is increased and, consequently, as the improbability of the event is increased.[15] However, Price conceded that in some circumstances the improbabilities may "affect the *credit* of testimony, or cause us to question its veracity" (1768, p. 417). Further, "The chief reason of the effect of improbabilities on our regard to testimony is, their tendency to influence the principles of deceit in the human mind" (1768, p. 420). Thus, it is not the improbability *per se* of a miraculous event that tends to diminish the value of the testimony but the fact that miracles are the kind of events that engage the passions of religion and the love of wonder and surprise. This concession is all that is needed for the above Bayesianized argument against miracles.

There is an appealing common sense core to this Bayesianized version of Hume's argument for his in-principle claim against religious miracles, and those who subscribe to the cautionary tale I sketched may be tempted to say that in this case Bayesianism is just common sense writ quantitatively. Giving in to that temptation would be a mistake, for there is nothing in Bayesianism *per se* that proves the in-principle claim. Those readers who know the dirty but by now fairly public secrets of Bayesianism will already know why this is so. Convincing those not already in the know is complicated only because the tent flying the banner "Bayesianism" has all manner of campers under its canvas, and each of the many sub-groups needs to be treated separately. Since a detailed examination of this matter would take me too far afield, I will restrict myself here to a classification which, though crude, is sufficient to illustrate the main points.[16]

Perhaps the largest group of campers call themselves personalists. For them, epistemology is conducted in terms of personal or subjective degrees of belief. These degrees of belief are, of course, required to satisfy the axioms of probability. Some but not all personalists also require (as assumed above) that when a person has a learning experience, the content of which is captured by a proposition, then her degrees of belief after the experience ought to equal her previous degrees of belief conditionalized on the learned proposition.[17] It should be evident without much argument that this personalist wing of Bayesianism allows for a wide latitude in degrees of belief, so wide in fact that some of the campers in this wing will agree with Hume's in-principle claim while others will have degrees of belief that satisfy (4) and, therefore, (1) for some miracles deemed to have religious significance.

Thomas Bayes himself was not a pure personalist. His goal was to explicate the notion of reasonable or rational inductive inference, which he thought to require constraints on the assignment of prior probabilities. An examination of Bayes' proposed constraints reveals a sophisticated though problematic theory.[18] But even supposing Bayes' method of imposing constraints were unproblematic, it would be unavailing in the present context. Bayes was concerned with the event or state of affairs of an objective chance parameter taking a certain value p. Assuming that p is known, the probabilities of outcomes of running the chance experiment a specified number of times can be calculated.[19] As a result, the posterior probability that p lies in a specified interval is fixed once the prior distribution over p is given. By contrast, the Bayesianized Hume who is trying to combat belief in miracles is concerned with a case where not only the prior probability $Pr(A/K)$ but also the likelihoods $Pr(a/A\&K)$ and $Pr(a/\neg A\&K)$ are up for grabs.

Those Bayesians who count themselves as objectivists would use frequency data to guide assignment of values to these factors. We have already seen that in the case of $Pr(A/K)$ the guidance cannot take the simple minded form of using the value of the frequency of A-type event in past experience. That frequency may be flatly zero, but it seems unwise to set $Pr(A/K) = 0$. For one thing, this leads to a probability measure that is not "strictly coherent." If probability is used as a guide to betting behavior, the agent whose degrees of belief are represented by such a measure can be induced to take a bet with the property that in no possible case can she win anything while in some possible case she will suffer a loss. As for the factors $Pr(a/A\&K)$ and $Pr(a/\neg A\&K)$ it is in principle possible to get relevant frequency data. But in the actual cases of alleged miracles of religious significance there is not enough undisputed data to give reliable estimates of the relevant frequencies. To overcome this difficulty one can try to reason by analogy with cases where the witnesses in question or ones like them testify as to the occurrence of a type of event that allows us to get undisputed and reliable frequency data. But there are no accepted rules for such analogical reasoning; and in keeping with the spirit of Bayesianism, such rules as we do fashion will be of a probabilistic form, involving priors and likelihoods about which the campers may differ. The chances of ending the regress that has started so that all would-be objectivists arrive at the same numbers seems dim at best.

The upshot is that every wing of the big tent of Bayesianism contains campers who meet all of the rationality constraints demanded by their brand of Bayesianism and yet who do not subscribe to Hume's in-principle claim that human testimony cannot have such a force as to establish a religious miracle.

This is hardly a surprising or unwanted conclusion. As any number of commentators have remarked, information about the probity of a particular witness or the weight of testimony from a number of independent witnesses may overcome initial doubts stemming from concerns about deception and deceit.[20] Bayes' theorem offers a simple explanation for the effect of independent witnessing coupled with minimal probity. Let a^n stand for the proposition that n witnesses have testified to the occurrence of the event. And for simplicity assume that each witness is as likely as any other to testify truly (i.e. $Pr(a/A\&K) = p$ for each) and is equally likely as any other to testify falsely (i.e. $Pr(a/\neg A\&K) = q$ for each). If we take independence to mean that $Pr(a^n/A\&K) = p^n$ and $Pr(a^n/\neg A\&K) = q^n$, Bayes' theorem yields

$$\Pr(A/a^n\&K) = \frac{1}{1 + \left[\frac{\Pr(\neg A/K)}{\Pr(A/K)} \left(\frac{q}{p}\right)^n\right]} \qquad (5)$$

Minimal probity means that p > q, with the result that (assuming Pr(*A*/*K*) > 0) as $n \to \infty$, $(q/p)^n \to 0$ and, hence, $\Pr(A/a^n\&K) \to 1$. The higher the probity ratio p/q, the faster certainty is reached. Or, to put the point in the manner of Charles Babbage (1838), no matter how small Pr(*A*/*K*) is as long as it is non-zero, it is possible to choose the number n of independent witnesses such that $\Pr(A/a^n\&K) > .5$.[21]

Turning from in-principle considerations to actual cases, what *is* disconcerting to common sense is that Bayesianism doesn't underwrite Hume's minimalist claim that no actual testimony for a religious miracle has ever amounted to a probability. My personal probabilities are in line with this claim. But the alignment is not dictated by the personalist form of Bayesianism or by any other workable form of Bayesianism of which I am aware. For those who find this negative result disconcerting, much worse is to follow.

4. *A Retreat That Becomes a Rout?*

Given the discussion of the preceding section we may assume that there are Bayesian agents and an *A* such that (1) holds, where *A* asserts the occurrence of an event which violates a well-confirmed (putative) law and which is of the type deemed to have religious significance. Having gotten this far we might as well simplify the subsequent discussion by further assuming that Pr(*A*/*a*&*K*) is so near one as makes no odds. Hume might have responded that even though a miracle has been proved, it "can never be proved so as to be a foundation of a system of religion." I am suggesting that the quoted phrase now be interpreted to mean that whatever religious significance the adherents of a religion want to attribute to the violation of the (putative) law, there are always alternative explanations of the violation that do not involve a divine being or similar notions at the core of the religion in question; and such alternatives can never be ruled out with certainty.

As already remarked above, Hume added a footnote in which he says that "A miracle may be accurately defined, *a transgression of a law of nature by a particular volition of the Deity, or by the interposition of some invisible agent*" (p. 115, n. 1). The charitable interpretation of this note is that Hume is anticipating the last ditch stand just sketched.[22] Without the charity, much of "Of Miracles" makes little sense. Part II discusses several examples of attested miracles from profane history, such as Tacitus' report of Vespasian who supposedly cured a blind man with spittle (p. 122), and Hume's hypothetical example of reports that Elizabeth I died on January 1, 1600 and that after being interred for a month, she reappeared and resumed the throne. (p.

128). In such cases the testimony is only to the occurrence of the alleged event and not to any divine origin. But nevertheless Hume thought that all of his strictures apply. About the Queen Elizabeth case he says that "I should only assert that it [her death] to have been pretended, and that it neither was, nor possibly could be real" (p. 128). He does go on to add that should this alleged miracle be ascribed to a new system of religion, "this very circumstance would be full proof of a cheat" (p. 129). But that is because

> As the violations of truth are more common in the testimony concerning religious miracles, than in that concerning any other matter of fact; this must diminish the authority of the former testimony, and make us form a general resolution, never to lend any attention to it ... (p. 129)

What then are we to say about the last ditch argument sketched above? Hume himself might have wished to use this argument, but a Bayesianized Hume cannot. To repeat, for a Bayesian, epistemology is not a matter of certainties but of greater and lesser probabilities.

The question now becomes whether the hypotheses at the core of a religion—for example the hypothesis D that there exists a divine being with specified characteristics—can be probabilified by the testimonial evidence to miracles. Since we have assumed that $\Pr(A/a\&K) = 1$, the question devolves onto the conditional probability $\Pr(D/A\&K)$. The first subquestion is whether A confirms or supports D in the sense that

$$\Pr(D/A\&K) > \Pr(D/K). \qquad (6)$$

Bayes' theorem gives as separately necessary and jointly sufficient conditions for (6): (i) $\Pr(D/K) > 0$ and (ii) $\Pr(A/D\&K) > \Pr(A/K)$ or, equivalently, (ii′) $\Pr(A/D\&K) > \Pr(A/\neg D\&K)$. Needless to say, the adherents of the religion will be eager to affirm (i). And for a miracle of the appropriate type they will surely think that A is more likely under the assumption that the divine being exists than it would be if He did not exist (see, for example, Swinburne 1979). For these adherents, (6) holds. Furthermore, there is no reason in principle why the accumulation of a series of miracles of the appropriate types cannot boost the probability of D above .5.

At least all of this is consistent with being a good Bayesian personalist. It may be uncongenial to those Bayesians who want all terms in Bayes' theorem to be grounded in objective frequencies. But such scruples would disqualify not only the probabilification of D but many of the non-statistical theoretical hypotheses of the advanced sciences.

The logical positivists and their fellow travellers thought that they had a different way of disqualifying D; namely, they labelled it as "cognitively meaningless." Initially they took verifiability, or falsifiability, or some combination of the two as the touchstone of the meaningful. But under pressure of various counterexamples they were forced to abandon this tack.[23] Reichenbach opted for a confirmability criterion by which a cognitively meaningful hypothesis is one which can have its probability raised or lowered by the evidence of observation. Reichenbach chose this criterion in part for its "overreaching" character.[24] He wanted to be a realist about the unobservables talked about in modern physics, and the Bayesian framework provides the means by which observation and experiment can generate degrees of belief in hypotheses about such entities. What I am suggesting is that the overreaching character of the method stretches much further than Reichenbach might have wanted and, indeed, it stretches into the religious realm. Reichenbach no doubt would have tried to draw the line using his frequency theory of probability. But nowhere does he give a workable frequency interpretation of the various terms of Bayes' theorem as applied to the theoretical hypotheses of modern science. Thus, Reichenbach's views on probability do not provide a way to block the reach of Bayesian inference to religious hypotheses that does not also block the reach to hypotheses of theoretical physics.

For those who want to be Bayesians and anti-religious at the same time, the situation I am pointing to can perhaps be partly defused by appealing to a Carnapian relativism.[25] There are (the story goes) a wide variety of linguistic/conceptual systems. It is the syntactical and semantic rules of a system that determine whether or not an expression is meaningful in that system. In some systems *D* and its like are well-formed and meaningful. And in these systems the Bayesian machinery can be used to discuss the confirmation of *D* by the testimony to miracles and by other evidence. In other systems *D* and its like will be ill-formed, either syntactically or semantically. For these systems the Bayesian machinery never gets into gear. The question, "Are we entitled to believe *D*?" must now be divided. It could be taken as an internal question, a question asked within a specified system. The answer is then as it is. If the system is one in which *D* is ill-formed, the answer is no because belief (or disbelief) does not properly attach to *D*. If the system is one in which *D* is well-formed, the answer is supplied for any person by cranking her version of the Bayesian machinery for the total available evidence. On the other hand, the question can be construed as an external question, a question asked from without all the systems. As such it can only be given sense as a query about which of the various systems one ought to use. Here the spirit of Carnap would reply that this query points to a pragmatic decision whose outcome will vary with the uses to which one chooses to put the system.[26]

I suspect that many religionists and anti-religionists alike would be unhappy with such a reading of their dispute. Certainly Price and Hume thought that they were engaged in a well-defined and heads-on cognitive dispute, not some relativist shuffle or a hassle over pragmatic factors. But I also think that this is the best anti-religionist Bayesians can do if they do not want their machinery turned against them.

To inject a personal note, my own Carnapian decision is in favor of a system within which religious hypotheses are counted as meaningful. And I agree with Swinburne (1979) that within such a system Bayesianism can be used to marshall inductive arguments in favor of these hypotheses. But my personal probabilities are not in line with Swinburne's ultimate conclusion that, all evidence considered, the balance of probability is in favor of the existence of the Christian deity. Of course, such disagreements arise for scientific as well as religious matters. What is striking about modern science is the objectivity of scientific belief in the sense of a tight consensus of degrees of beliefs regarding core scientific hypotheses. I would contend, however, that there is no distinctively Bayesian explanation of this consensus and that one must look elsewhere, such as to evolutionary or sociological factors, for an explanation.[27]

5. *Conclusion*

Commentators on Hume's miracle argument tie themselves in knots trying to craft a definition of 'miracle'. I suspect that there is no single, simple definition that answers to all of the demands that are put on this concept. But we do not have to settle on any one definition to see the holes in Hume's argumentation.

1) If the argumentation of Part I of Hume's essay works, then it works against a miracle defined as a violation of a well confirmed and here-to-fore unviolated lawlike regularity. Only a crude view of induction and probability could have led to such a result. The more sophisticated view of inductive reasoning developed by the Rev. Thomas Bayes is an antidote, but since Bayes' essay was not published until 1763, Hume could not have availed himself of this approach.

2) Whatever the niceties of the definition of 'miracle', it is abundantly clear that for Hume a resurrection counted as a clear case of a miracle. It would be surprising if it were otherwise since the 18th century debate over miracles as a basis for the Christian revelation focused on this case. Moreover, Hume's concentration on testimonial evidence is explained by the fact that the belief in the resurrection of Christ depended on the testimony of the Apostles. Several claims can be made about such a miracle. (a) Because of man's love of wonder and the passions of religion, one should be cautious about accepting testimony to the occurrence of such a miracle. (b) No testimony in the actual historical record establishes the credibility of such a miracle. (c) It is not possible in principle to establish the credibility of such a miracle by means of testimonial evidence. (d) Evidence for such a miracle (testimonial or otherwise) cannot serve to establish the credibility of doctrines at the core of popular religions. Claim (a) is a platitude that does not require the support of any philosophical argumentation, Bayesian or otherwise. Claim (c) is a non-starter, at least by Bayesian lights. Claim (b) is not vindicated by

Bayesianism. Bayesianism can be used to show how to argue for (b); but it can also be used to show how to argue against (b).[28] A similar point holds for (d), but this point deserves a more detailed discussion.

3) Some commentators want to define a miracle as an event which defies any natural explanation and which, therefore, demands a religious or at least a supernatural explanation. Hume seems to presuppose such a view when he states his contrary miracles argument. Suppose that the proposition M_i, i = 1, 2, ... , asserting the occurrence of a miracle of type i, is evidence for religious doctrine D_i in the strong sense that M_i could only be true if D_i is true. Then if the D_i are pairwise incompatible (as will be the case if they belong to competing religions), then no two of the miracles could have occurred and, consequently, any evidence for M_j tends to cancel evidence for M_k when $j \neq k$. As Hume puts it, if a miracle has the force to establish a particular system of religion, "so it has the same force, though more indirectly, to overthrow every other system ..." (p. 121). From the Bayesian perspective this argumentation presupposes a crude view of the evidential role of miracles since it fails to allow that miracles can serve as evidence for a religion in the sense of raising the probability of the truth of its doctrines without serving as proof positive. Of course, one now has to face the problem that on the more sophisticated conception of the evidential import of miracles, evidence for the resurrection of Jesus of Nazareth will not serve as unequivocal evidence for the Christian religion. But an exactly similar situation obtains in the sciences where there are often many competing theories and where the evidence of observation and experiment rarely accords with only one of the theories. In both situations, the evidence has to be assessed in terms of degrees of belief, not in terms of "proofs." Here a Hume who has donned the mantle of Bayesianism might maintain that there is nevertheless an important in-principle distinction between the two cases: that whereas in science the accumulating evidence can lead to a firm consensus about which of the competing theories is probably true, the evidence for miracles is incapable of engendering a rational consensus about which religion is probably true. Such a scepticism about the theological doctrines of competing religions is compatible with Hume's declaration that considerations of design make it reasonable to believe in the existence of *a* deity. Of course, some argument for this more sophisticated form of religious scepticism is needed. The argument, if it could be made, would be more interesting than anything found in "Of Miracles."[29]

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NOTES

1. Bayes' essay was published in 1763 under the title "An Essay Towards Solving a

Problem of the Doctrine of Chances," *Philosophical Transactions of the Royal Society (London) 53*: 370-418. It is reprinted in *Biometrika 45* (1958): 296-315. For the dating of Bayes' essay, see Dale (1986).

2. Price gives a reference to Bayes' essay but does not mention Bayes by name; see Price (1768, p. 395).

3. See Klibansky and Mosser (1954, p. 234).

4. There are many different versions of "Bayes' theorem." The one used here states that

$$Pr(H/E\&K) = \frac{Pr(H/K) x Pr(E/H\&K)}{Pr(E/K)}$$

Sometimes the principle of total probability is used to rewrite the denominator on the right hand side as $Pr(E/H\&K) x Pr(H/K) + Pr(E/\neg H\&K) x Pr(\neg H/K)$. The reader is invited to think of H as a hypothesis at issue; K as the background knowledge; and E as the additional evidence. $Pr(H/E\&K)$ is called the posterior probability of H. $Pr(H/K)$ and $Pr(E/H\&K)$ are respectively called the prior probability of H and the (posterior) likelihood of E.

5. All page references are to the Selby-Bigge/Nidditch edition of the 1777 posthumous edition of the *Enquiry*.

6. See the discussion in section 3 below.

7. Sobel (1987) points out that the problem of "old evidence" rears its ugly head in this context. I will ignore the problem here although I think that it poses one of the most difficult challenges facing Bayesian confirmation theory; see my 1989.

8. Price paraphrased Hume's Maxim as asserting that "no testimony should engage our belief, except the improbability of the falsehood of it is greater than that in the event it attests." (1768, p. 405) Price explicitly states that (in our notation) the improbability of the event means $Pr(\neg A/K)$. His subsequent discussion leaves in doubt his interpretation of the improbability the falsehood of the testimony. One reading suggested by his examples is that he took this term to mean $Pr(\neg A/a\&K)$. This turns Hume's Maxim into an absurdity since the unless clause comes to the condition that $Pr(\neg A/K) < Pr(\neg A/a\&K)$ or, equivalently, that $Pr(A/K) > Pr(A/a\&K)$, which says that the testimony disconfirms A.

9. A superficial reading of Babbage might suggest that he is putting forward the Gillies-Sobel condition that $Pr(A/K) > Pr(a\&\neg A/K)$. However, his formulas do not make sense unless interpreted in terms of the conditional probabilities $Pr(a/A\&K)$ and $Pr(a/\neg A\&K)$. See Babbage (1838, pp. 196-197).

10. See Hendel (1955, p. 137, n. 11).

11. Sobel (1987) considers the possibility that $Pr(A/K)$ be given a non-zero but infinitesimal value. On this suggestion, see Owen (1987) and Dawid and Gillies (1989).

12. Using Bayes's suggested prior probability distribution, it follows that if n trials are run and the type of event in question occurs in each of them, then the probability that the event will occur in the next trial is $(n + 1)/(n + 2)$. Bayes's prior makes the probability of the hypothesis that the event will occur for all (countably infinite) future trials flatly 0, which strikes some as being overly skeptical.

13. This contention is challenged directly by Price; see the following section.

14. In modern parlance, Hume subscribed to Reichenbach's "straight rule" of induction which violates strict coherence; see section 4 below.

15. It is instructive for the reader to apply formula (4) to this case, taking A to be the proposition that ticket #11,423 was drawn in the lottery and a as the proposition that a report to this effect appeared in the newspaper. Dawid and Gillies (1989) analyze the crucial term $\Pr(a/\neg A\&K)$ on the assumption that if the newspaper makes a mistake and prints an incorrect number, it is no more likely to print one number than another. The reader may also want to consider the, perhaps, more plausible assumption that if the newspaper prints an incorrect number, the most likely scenario is that it has reversed two digits or mistranscribed one of the digits.

16. More details can be found in my 1992.

17. As an example of someone who rejects conditionalization, see van Fraassen (1989).

18. For details, see my 1990 and 1992.

19. The calculation assumes what David Lewis (1980) calls the "Principal Principle." Very roughly the idea is that if I know for sure that, say, the objective probability of heads on a flip of a coin is p, then my subjective probability that the next flip ought to be heads is p, regardless of what else I know about the number of heads in past flips.

20. Hume himself says as much in discussing the case of an eclipse which contravenes Newtonian laws (see Selby-Bigge 1975, pp. 127-128). But immediately thereafter in discussing the hypothetical case of a resurrection of Queen Elizabeth, he says in effect that no amount of testimony would convince him that "so signal a violation of the laws of nature" had taken place. Hume gives no principled way to distinguish the cases. But presumably he thinks that in the latter case the witnesses cannot be as independent as in the eclipse case. And presumably the quasi-religious nature of the latter miracles raises a greater suspicion of "knavery and folly."

21. Babbage's calculation suffers from the fact that he assumes that $q = 1 - p$ (see Note E, pp. 186-203 of his 1838). Babbage does not refer to Bayes but instead relies on the work of Laplace, Poisson, and Demorgan. I am grateful to Sandy Zabell for calling Babbage's work to my attention.

22. Here I part company with Flew's interpretation; see Flew (1985, pp. 4-7).

23. The story is too well known to need repetition here; for a retelling, see Hempel (1965).

24. In *Experience and Prediction* Reichenbach wrote that "The probability theory of meaning ... allows us to maintain propositions as meaningful which concern facts outside the domain of immediately given verifiable facts; it allows us to pass beyond the domain of given facts. This *overreaching* character of probability inferences is the basic method of the knowledge of nature." (1961, p. 127). Religionists may claim that it is also the basic method of the knowledge of God.

25. I have in mind Carnap's (1934) and (1952).

26. I do not know whether Carnap himself would have wanted to apply the internal/external question apparatus to questions about the status of religious beliefs. But I note that he did apply it to questions about physicalism and mind-body identity; see Carnap (1963).

27. See Ch. 6 of my 1992. Powerful merger of opinion results can be proven within the Bayesian framework. However since these results refer to an infinite limit, they fail to

explain the actual consensus beliefs that arise in the medium and short runs. Moreover, these results are unavailable when the hypotheses at issue are underdetermined by the possible evidence.

28. Some will rejoice in such a conclusion. Others will brood that Bayesianism cannot be the whole story of ampliative inference. Banner (1990) tries to overcome some of the perceived weaknesses of Bayesianism through a methodology based around inference to the best explanation. He then argues that religious beliefs can be justified using this methodology. I share van Fraassen's (1989) misgivings about inference to the best explanation.

29. Thanks are due to Donald Gillies, Howard Sobel, and Sandy Zabell for helpful comments on an earlier draft of this paper.

REFERENCES

Babbage, C.
1838. *The Ninth Bridgewater Treatise*, 2d ed. (London: Frank Cass and Co., 1967).

Banner, M. C.
1990. *The Justification of Science and the Rationality of Religious Belief* (Oxford: Clarendon Press).

Carnap, R.
1934. *Logische Syntax der Sprache* (Wein: Julius Springer).
1952. "Empiricism, Semantics, and Ontology," in L. Linsky (ed.), *Semantics and the Philosophy of Language* (Urbana: University of Illinois Press).
1963. "Herbert Feigl on Physicalism," in P.A. Schilpp (ed.), *The Philosophy of Rudolf Carnap* (La Salle: Open Court).

Dale, A.I.
1986. "A Newly Discovered Result of Thomas Bayes'," *Archive for the History of the Exact Sciences 35*: 101-113.

Dawid, P. and Gillies, D.
1989. "A Bayesian Analysis of Hume's Argument Concerning Miracles," *Philosophical Quarterly 39*: 57-63.

Earman, J.
1989. "Old Evidence, New Theories: Two Unsolved Problems of Bayesian Confirmation Theory," *Pacific Philosophical Quarterly 70*: 323-340.
1990. "Bayes' Bayesianism," *Studies in the History and Philosophy of Science 21*: 351-370.
1992. *Bayes or Bust? A Critical Examination of Bayesian Confirmation Theory* (Cambridge: MIT Press/Bradford Books).

Flew, A. (ed.)
1985. *Of Miracles* (La Salle: Open Court).

Gillies, D.
1991. "A Bayesian Proof of a Humean Principle," *British Journal for the Philosophy of Science 42*: 255-256.
Hempel, C.G.
1965. "Empiricist Criteria of Cognitive Significance: Problems and Changes," in *Aspects of Scientific Explanation* (New York: Free Press).
Hendel, C.W. (ed.)
1955. *An Enquiry Concerning Human Understanding* (Indianapolis: Bobbs-Merrill).
Klibansky, R. and Mosser, E.C. (eds.)
1954. *New Letters of David Hume* (Oxford: Clarendon Press).
Owen, D.
1987. "Hume *versus* Price on Miracles and Prior Probabilities: Testimony and the Bayesian Calculation," *Philosophical Quarterly 37*: 187-202.
Price, R.
1768. *Four Dissertations* 2d ed. (London: A Millar and T. Caddel).
Reichenbach, H.
1961. *Experience and Prediction* (Chicago: University of Chicago Press).
Selby-Bigge, L.A. and Nidditch, P.H. (eds.)
1975. *Enquiries Concerning Human Understanding and Concerning the Principles of Morals* (Oxford: Clarendon Press).
Sobel, J. H.
1987. "On the Evidence of Testimony for Miracles: A Bayesian Interpretation of David Hume's Analysis," *Philosophical Quarterly 37*: 166-186.
1991. "Hume's Theorem on Testimony Sufficient to Establish a Miracle," *Philosophical Quarterly 41*: 229-237.
Swinburne, R.
1979. *The Existence of God* (Oxford: Clarendon Press).
van Fraassen, B.C.
1989. *Laws and Symmetry* (Oxford: Clarendon Press).