

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# ALGEBRA II

Wednesday, June 21, 2023 — 9:15 a.m. to 12:15 p.m.,

## MODEL RESPONSE SET

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**Question 25**

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**25** The business office of a local college wishes to determine the methods of payment that will be used by students when buying books at the beginning of a semester. Explain how the office can gather an appropriate sample that minimizes bias.

Pick random names from a list of students  
and ask them how they will pay.

**Score 2:** The student gave a complete and correct response.

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**Question 25**

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**25** The business office of a local college wishes to determine the methods of payment that will be used by students when buying books at the beginning of a semester. Explain how the office can gather an appropriate sample that minimizes bias.

The college should take a random  
Sample Survey of students walking onto  
Campus.

**Score 2:** The student gave a complete and correct response.

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**Question 25**

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**25** The business office of a local college wishes to determine the methods of payment that will be used by students when buying books at the beginning of a semester. Explain how the office can gather an appropriate sample that minimizes bias.

The office can make a survey at the mall or a public place and this will get them an unbiased information because they are randomly choosing people.

**Score 1:** The student did not survey an appropriate sample.

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**Question 25**

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**25** The business office of a local college wishes to determine the methods of payment that will be used by students when buying books at the beginning of a semester. Explain how the office can gather an appropriate sample that minimizes bias.

ASK a certain amount of students in each grade  
how they make their payments when buying books.

**Score 1:** The student did not describe a random selection process.

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**Question 25**

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**25** The business office of a local college wishes to determine the methods of payment that will be used by students when buying books at the beginning of a semester. Explain how the office can gather an appropriate sample that minimizes bias.

Take a survey

**Score 0:** The student did not show enough correct work to receive any credit.

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**Question 25**

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**25** The business office of a local college wishes to determine the methods of payment that will be used by students when buying books at the beginning of a semester. Explain how the office can gather an appropriate sample that minimizes bias.

They can test it out  
by samples

**Score 0:** The student did not show enough correct work to receive any credit.

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**Question 26**

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**26** Determine the solution of  $\sqrt{3x + 7} = x - 1$  algebraically.

$$3x + 7 = x^2 - 2x + 1$$

$$0 = x^2 - 5x - 6$$

$$0 = (x - 6)(x + 1)$$

$$x = 6$$

**Score 2:** The student gave a complete and correct response.



Question 26

26 Determine the solution of  $\sqrt{3x+7} = x-1$  algebraically.

$$(\sqrt{3x+7})^2 = (x-1)^2$$

$$3x+7 = x^2 - 2x + 1$$

$$x^2 - 5x - 6 = 0$$

$$(x-1)(x-1)$$

$$x^2 - x - x + 1$$

$$x^2 - 2x + 1$$

$$x = \frac{5 \pm \sqrt{(5)^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 - (-24)}}{2}$$

$$x = \frac{5 \pm 7}{2}$$

$$x = 6$$

$$x = -1$$

extraneous  
root

$$\sqrt{3(6)+7} = 6-1$$

$$5 = 5$$

$$\sqrt{3(-1)+7} = -1-1$$

$$2 \neq -2$$

**Score 2:** The student gave a complete and correct response.

**Question 26**

26 Determine the solution of  $\sqrt{3x+7} = x-1$  algebraically.

$$(\sqrt{3x+7})^2 = (x-1)^2 \quad (x-1)(x-1)$$

$$\begin{array}{r} 3x+7 = x^2 - 2x+1 \\ -3x-7 \quad -3x-7 \\ \hline \end{array}$$

$$0 = x^2 - 5x - 6$$

$$(x-6)(x+1)$$

$$\boxed{x=6} \quad \boxed{x=-1}$$

**Score 1:** The student did not reject  $-1$ .

Question 26

26 Determine the solution of  $\sqrt{3x+7} = x-1$  algebraically.

$$(\sqrt{3x+7})^2 = (x-1)^2 \quad (x-1)(x-1)$$
$$x^2 - 2x + 1$$

$$3x+7 = x^2 - 2x + 1$$

$$-x^2 + 5x + 6 = 0$$

$$25 - 4(-1)(6)$$

$$\frac{-5 \pm 7}{-2} = \{-1, 6\}$$

$$\sqrt{3(-1)+7} = -1-1$$

$$\sqrt{-4} = -2$$

$$\pm 2 = -2 \quad \checkmark$$

$$\sqrt{3(6)+7} = 6-1$$

$$\pm 5 = 5 \quad \checkmark$$

$$\{-1, 6\}$$

**Score 1:** The student incorrectly found the square root of 4.

Question 26

26 Determine the solution of  $\sqrt{3x+7} = x-1$  algebraically.

|       |      |      |
|-------|------|------|
| $x$   | $1$  |      |
| $x^2$ | $1x$ | $x$  |
| $-6x$ | $-6$ | $-6$ |

$\downarrow$   
 $-5x$

$$\begin{array}{r} 3x+7 = x^2-2x+1 \\ -3x-7 \quad -3x-7 \\ \hline 0 = x^2-5x-6 \end{array}$$

|              |             |
|--------------|-------------|
| $(x+1)(x-6)$ |             |
| $x \neq -1$  | $x \neq -6$ |

unconstitutional

|       |       |      |
|-------|-------|------|
| $x$   | $-1$  |      |
| $x^2$ | $-1x$ | $x$  |
| $-1x$ | $1$   | $-1$ |

**Score 1:** The student made an error finding  $x$ , but then rejected correctly.

Question 26

26 Determine the solution of  $\sqrt{3x+7} = (x-1)$  algebraically.

$$3x+7 = (x-1)(x-1)$$

$$3x+7 = x^2 - x - x + 1$$

$$3x+7 = x^2 + 2x + 1$$

$$-3x - 7$$

$$x^2 - x - 6$$

$$(x-3)(x+2)$$

$$x-3=0 \quad | \quad x+2=0$$

|       |        |
|-------|--------|
| $x=3$ | $x=-2$ |
|-------|--------|

**Score 0:** The student made a computational error and did not reject correctly.

Question 26

26 Determine the solution of  $\sqrt{3x+7} = x-1$  algebraically.

$$\sqrt{3x+7} = x-1$$
$$3x+7 = (x-1)(x-1)$$

$$\sqrt{3x+7} = x-1$$
$$+1 \quad +1$$

$$(\sqrt{3x+7} + 1)^2 = (x)^2$$

$$3x+7+1 = x^2$$

$$3x+8 = x^2$$
$$-3x \quad -3x$$

$$8 = x^2 - 3x$$
$$-8 \quad -8$$

$$x^2 - 3x - 8$$

$$+ 2$$
$$6 \quad 1$$

**Score 0:** The student did not do enough correct work to receive any credit.

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**Question 27**

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**27** The population of bacteria,  $P(t)$ , in hundreds, after  $t$  hours can be modeled by the function  $P(t) = 37e^{0.0532t}$ . Determine whether the population is increasing or decreasing over time. Explain your reasoning.

The population is increasing over time. If you graph the equation, as the  $x$  values increase, the  $y$  values increase in greater intervals.

**Score 2:** The student gave a complete and correct response.

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**Question 27**

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27 The population of bacteria,  $P(t)$ , in hundreds, after  $t$  hours can be modeled by the function  $P(t) = 37e^{0.0532t}$ . Determine whether the population is increasing or decreasing over time. Explain your reasoning.

$$e^{.0532} \approx 1.0546 \text{ is greater than } 1 \text{ so}$$

Increasing

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**Score 2:** The student gave a complete and correct response.



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**Question 27**

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27 The population of bacteria,  $P(t)$ , in hundreds, after  $t$  hours can be modeled by the function  $P(t) = 37e^{0.0532t}$ . Determine whether the population is increasing or decreasing over time. Explain your reasoning.

Increasing - the values steadily increase

**Score 1:** The student gave an incomplete explanation.

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**Question 27**

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27 The population of bacteria,  $P(t)$ , in hundreds, after  $t$  hours can be modeled by the function  $P(t) = 37e^{0.0532t}$ . Determine whether the population is increasing or decreasing over time. Explain your reasoning.

It increase because its getting multiplied

**Score 0:** The student did not give enough of an explanation to receive any credit.

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**Question 27**

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**27** The population of bacteria,  $P(t)$ , in hundreds, after  $t$  hours can be modeled by the function  $P(t) = 37e^{0.0532t}$ . Determine whether the population is increasing or decreasing over time. Explain your reasoning.

The population of the bacteria  
will decrease because of  
the exponent.

**Score 0:** The student stated decrease and wrote an incomplete explanation.

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**Question 28**

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**28** The polynomial function  $g(x) = x^3 + ax^2 - 5x + 6$  has a factor of  $(x - 3)$ . Determine the value of  $a$ .

$$0 = 27 + 9a - 15 + 6$$

$$0 = 18 + 9a$$

$$a = -2$$

**Score 2:** The student gave a complete and correct response.

**Question 28**

28 The polynomial function  $g(x) = x^3 + ax^2 - 5x + 6$  has a factor of  $(x - 3)$ . Determine the value of  $a$ .

$$g(3) = (-3)^3 + a(-3)^2 - 5(-3) + 6$$

$$0 = -27 + 9a + 15 + 6$$

$$0 = 9a - 6$$

$$\frac{6}{9} = \frac{9a}{9}$$

$$\frac{2}{3} = a$$

**Score 1:** The student used  $-3$  for  $x$ .

**Question 28**

28 The polynomial function  $g(x) = x^3 + ax^2 - 5x + 6$  has a factor of  $(x - 3)$ . Determine the value of  $a$ .

$$g(x) = x^3 + ax^2 - 5x + 6$$

$$x^2(x+a) \parallel (-5x+6)$$

$$x+a = -5x+6$$

$$a = -2$$

**Score 1:** The student found the correct answer with no correct work.

**Question 28**

28 The polynomial function  $g(x) = x^3 + ax^2 - 5x + 6$  has a factor of  $(x - 3)$ . Determine the value of  $a$ .

$$\begin{aligned} 3 &= 3^3 + a3^2 - 5(3) + 6 & 3 &= 3^3 + a3^2 - 5(3) + 6 \\ 3 &= 27 + a9 - 15 + 6 & 3 &= 27 + a9 - 15 + 6 \\ \frac{3}{-27} & & 3 &= 27 + a9 - 9 \\ -24 &= a9 - 15 + 6 & & \\ + 15 & & & \\ -9 &= a9 + 6 & & \end{aligned}$$

**Score 0:** The student did not show enough correct work to receive any credit.





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**Question 29**

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29 Write a recursive formula for the sequence 189, 63, 21, 7, ... .

$$A_n = A_{n-1} \div 3.$$

$$A_1 = 189$$

**Score 2:** The student gave a complete and correct response.

Question 29

29 Write a recursive formula for the sequence 189, 63, 21, 7, ...

$$\frac{189}{3} \rightarrow \frac{63}{3} \rightarrow \frac{21}{3}$$

$$r = \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}$$

$$a_n = 189 \left( \frac{1}{3} \right)^{n-1}$$

**Score 1:** The student wrote an explicit formula.

Question 29

29 Write a recursive formula for the sequence 189, 63, 21, 7, ... .

$$a_n = \frac{a_{n-1}}{3}$$

$$\frac{189}{?} = 63$$

$$\frac{189}{3} = 63$$

$$\frac{63}{3} = 21$$

$$\frac{21}{3} = 7$$

**Score 1:** The student did not state  $a_1$ .

Question 29

29 Write a recursive formula for the sequence 189, 63, 21, 7, ...

$\underbrace{\hspace{2cm}}$   
-3

$$f(1) = 189$$

$$f(2) = 63$$

$$f(x-1) = x \div 3$$

**Score 0:** The student did not show enough correct work to receive any credit.

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**Question 29**

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29 Write a recursive formula for the sequence 189, 63, 21, 7, ... .

$$a_n = a_1 (3)^{n-1}$$

$$a_n = 189 (3)^{n-1}$$

**Score 0:** The student wrote an incorrect explicit formula.

Question 30

30 Solve algebraically for  $x$  to the nearest thousandth:

$$2e^{0.49x} = 15$$

$$\frac{2e^{0.49x}}{2} = \frac{15}{2}$$

$$e^{0.49x} = 7.5$$

$$\frac{0.49x \ln e}{\ln e} = \frac{\ln 7.5}{\ln e}$$

$$= 1$$

$$\frac{0.49x}{0.49} = \frac{2.014903021}{0.49}$$

$$x = 4.112046981$$

$$x \approx 4.112$$

**Score 2:** The student gave a complete and correct response.

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**Question 30**

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30 Solve algebraically for  $x$  to the *nearest thousandth*:

$$\frac{2e^{0.49x}}{2} = \frac{15}{2}$$

$$e^{0.49x} = 7.5$$

$$.49x = \log_e 7.5$$

$$x = \frac{\log_e 7.5}{.49}$$

$$x = 4.112$$

**Score 2:** The student gave a complete and correct response.

Question 30

30 Solve algebraically for  $x$  to the nearest thousandth:

$$\frac{2e^{0.49x}}{2} = \frac{15}{2}$$

$$\ln e^{.049x} = \ln 7.5$$

$$\frac{.049x}{.049} = \frac{\ln 7.5}{.049}$$

$$x = 41.120$$

**Score 1:** The student made a transcription error.



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**Question 30**

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30 Solve algebraically for  $x$  to the *nearest thousandth*:

$$2e^{0.49x} = 15$$

$$\ln(15) = .49x$$

$$\frac{5.416100402}{.49} = \frac{.49x}{.49}$$

$$11.0532653 = x$$

$$x = 11.053$$

**Score 1:** The student multiplied by 2 instead of dividing by 2.

Question 30

30 Solve algebraically for  $x$  to the nearest thousandth:

$$\frac{2e^{0.49x}}{2} = \frac{15}{2}$$

$$\frac{e^{0.49x}}{e} = \frac{7.5}{e}$$

$$\frac{0.49x}{0.49} = \frac{2.759095}{0.49}$$

$$X = 5.63080$$

$$X \approx 5.63$$

**Score 0:** The student did not show enough correct work to receive any credit.

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**Question 30**

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**30** Solve algebraically for  $x$  to the *nearest thousandth*:

$$\ln(2e^{0.49x}) = \ln(15)$$

$$x = 4,113$$

**Score 0:** The student did not show enough correct work to receive any credit.

**Question 31**

31 For all values of  $x$  for which the expression is defined, write the expression below in simplest form.

$$\frac{2x^3 + x^2 - 18x - 9}{3x - x^2}$$

$$\frac{x^2(2x+1) - 9(2x+1)}{x(3-x)}$$
$$= \frac{(x+3)\cancel{(x-3)}(2x+1)}{x\cancel{(3-x)}}$$
$$= \frac{(x+3)(2x+1)}{x}$$

**Score 2:** The student gave a complete and correct response.

Question 31

31 For all values of  $x$  for which the expression is defined, write the expression below in simplest form.

$$\frac{2x^3 + x^2 - 18x - 9}{3x - x^2}$$

$$\frac{x^2(2x+1) - 9(x+1)}{-x(3+x)}$$

$$\frac{(x^2 - 9)(2x+1)}{-x(3+x)}$$

$$\frac{(x+3)(x-3)(2x+1)}{-x(3+x)}$$

$$\frac{(2x+1)(x-3)}{-x}$$

**Score 1:** The student made a factoring error.

Question 31

31 For all values of  $x$  for which the expression is defined, write the expression below in simplest form.

$$\frac{2x^3 + x^2 - 18x - 9}{3x - x^2}$$

$$\frac{2x^3 + x^2 - 18x - 9}{x(3-x)}$$

$$\begin{array}{r} -2x+5 \\ \hline -x^2+3x \overline{) 2x^3+x^2-18x-9} \\ \underline{-(-2x^3+6x^2)} \quad \downarrow \quad \downarrow \\ -5x^2-18x \\ \underline{-(-5x^2+15x)} \\ -33-9 \end{array}$$

$$2x^3 + x^2 - 18x - 9$$

$$(2x+1)(x^2-9)$$

$$2x^3 - 18x + x^2 - 9$$

$$\frac{(2x+1)(x+3)(x-3)}{x(3-x)}$$

**Score 1:** The student did not leave the answer in simplest form.

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**Question 31**

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31 For all values of  $x$  for which the expression is defined, write the expression below in simplest form.

$$\frac{2x^3 + x^2 - 18x - 9}{3x - x^2}$$

$$x^2(2x+1) \mid -9(2x+1)$$

$$(x^2 - 9)(2x+1)(2x+1)$$

$$\frac{(x+3)(x-3)(2x+1)}{3x-x^2}$$

**Score 1:** The student only factored the numerator correctly.

**Question 31**

31 For all values of  $x$  for which the expression is defined, write the expression below in simplest form.

$$\frac{2x^3 + x^2 - 18x - 9}{3x - x^2}$$

$$\frac{x^2(2x) - 9(2x)}{3x - x^2}$$

$$\frac{(x^2 - 9)}{-x^2 + 3x}$$

$$\frac{(x+3)(x-3)}{-x^2 + 3x}$$

**Score 0:** The student did not show enough correct work to receive any credit.



Question 31

31 For all values of  $x$  for which the expression is defined, write the expression below in simplest form.

$$\frac{(2x^3 + x^2 - 18x - 9)}{3x - x^2}$$

$$\frac{x^2(2x) - 9(2x)}{x(3-x)}$$

$$x(3-x)$$

$$\frac{(x^2-9)(2x)}{x(3-x)}$$

$$x(3-x)$$

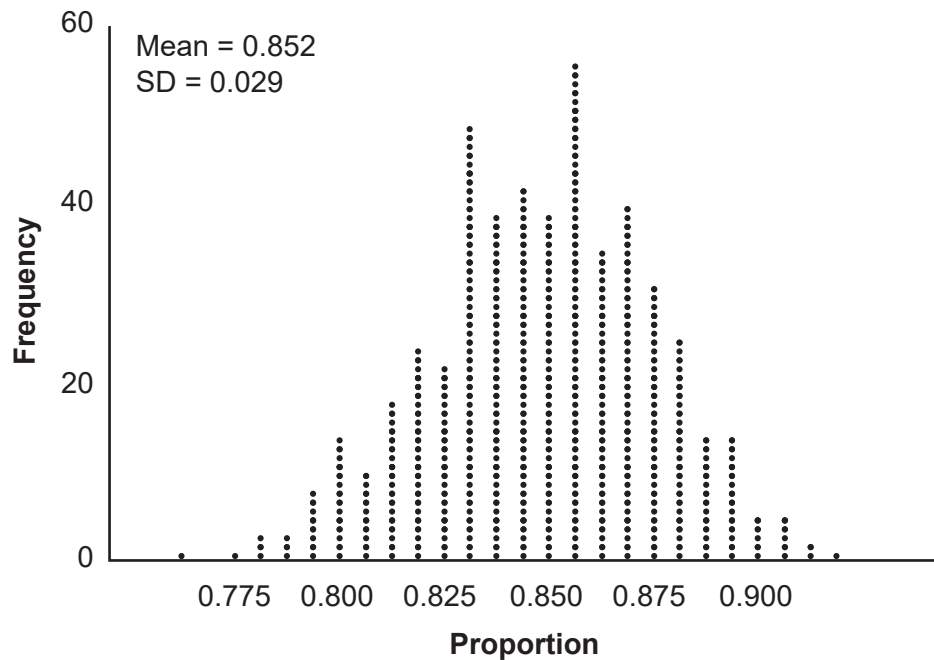
$$\frac{(x+3)(x-3)(2x)}{x(3-x)}$$

$$\frac{(-x+3)(2x)}{x}$$

**Score 0:** The student made multiple errors.

### Question 32

32 An app design company believes that the proportion of high school students who have purchased apps on their smartphones in the past 3 months is 0.85. A simulation of 500 samples of 150 students was run based on this proportion and the results are shown below.



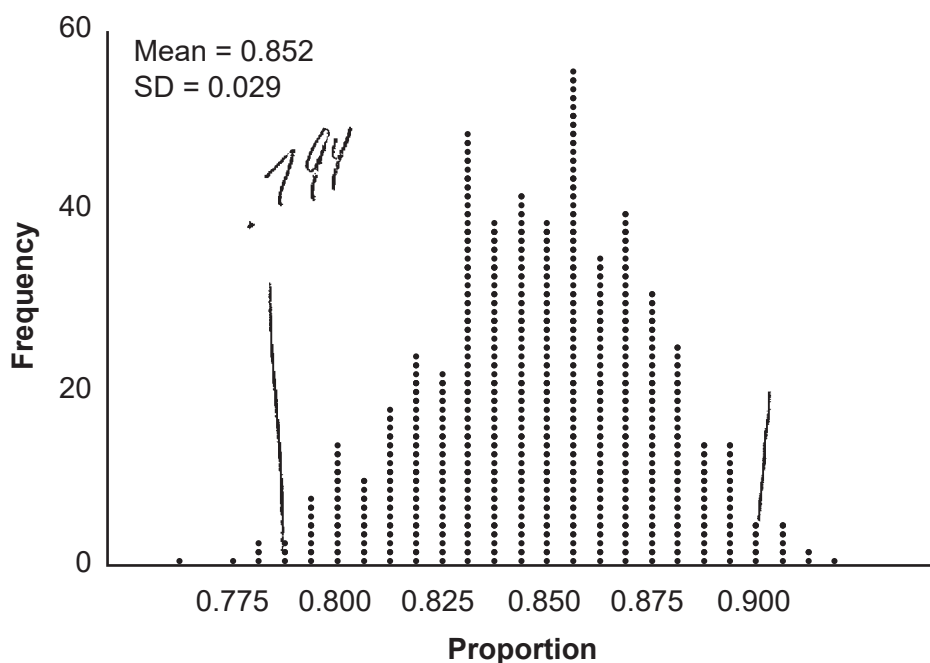
Suppose a sample of 150 students from your high school showed that 88% of students had purchased apps on their smartphones in the past 3 months. Based on the simulation, would the results from your high school give the app design company reason to believe their assumption is *incorrect*? Explain.

No b/c the high school's sample falls within  
the margin of error interval: 91% - 79.4%

**Score 2:** The student gave a complete and correct response.

**Question 32**

32 An app design company believes that the proportion of high school students who have purchased apps on their smartphones in the past 3 months is 0.85. A simulation of 500 samples of 150 students was run based on this proportion and the results are shown below.



Suppose a sample of 150 students from your high school showed that 88% of students had purchased apps on their smartphones in the past 3 months. Based on the simulation, would the results from your high school give the app design company reason to believe their assumption is *incorrect*? Explain.

$$2(0.029) = .058$$

$$0.852 + .058 = .91$$

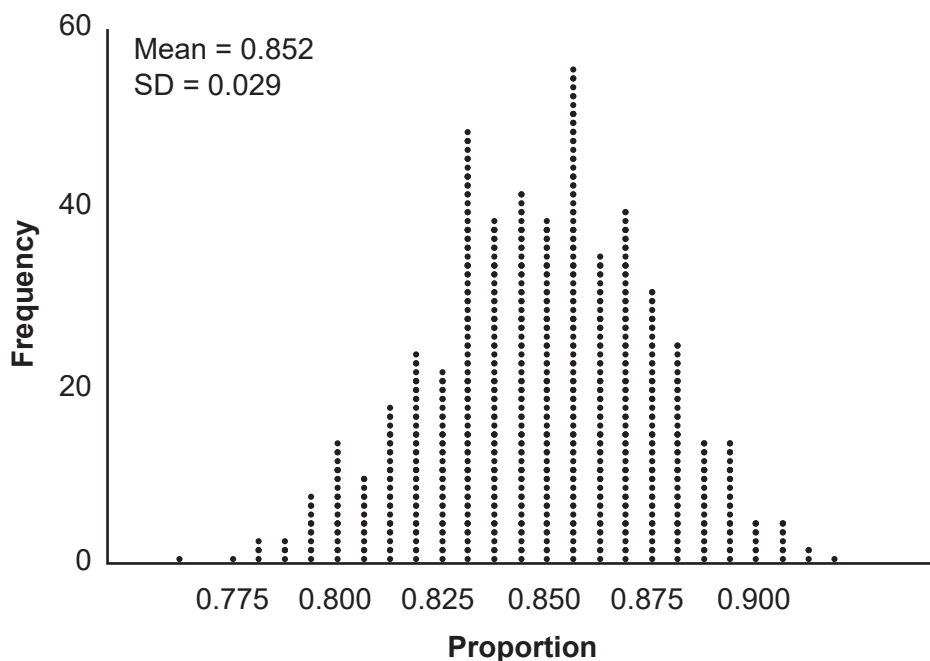
$$0.852 - .058 = .794$$

[.794, .91] the results would not give the company reason to believe that their assumption is not correct because 88% falls in the confidence interval.

**Score 2:** The student gave a complete and correct response.

**Question 32**

32 An app design company believes that the proportion of high school students who have purchased apps on their smartphones in the past 3 months is 0.85. A simulation of 500 samples of 150 students was run based on this proportion and the results are shown below.



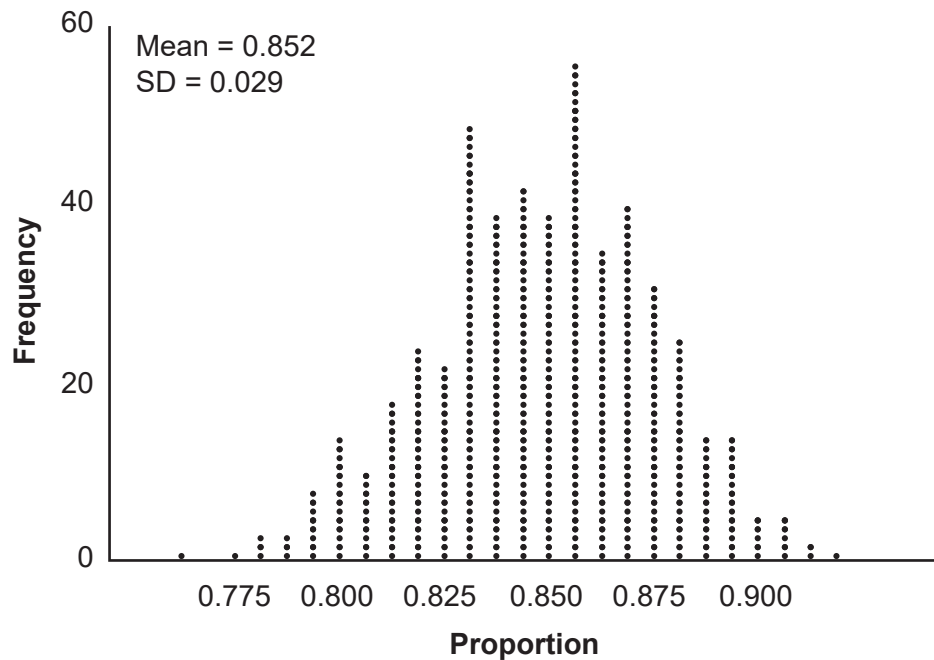
Suppose a sample of 150 students from your high school showed that 88% of students had purchased apps on their smartphones in the past 3 months. Based on the simulation, would the results from your high school give the app design company reason to believe their assumption is *incorrect*? Explain.

$Mean \pm SD$   
 $0.852 \pm 2(0.029)$   
 $\leftarrow .794 \quad \rightarrow .91$   
 NO  
 it IS  
 correct

**Score 1:** The student wrote an incomplete explanation.

### Question 32

32 An app design company believes that the proportion of high school students who have purchased apps on their smartphones in the past 3 months is 0.85. A simulation of 500 samples of 150 students was run based on this proportion and the results are shown below.



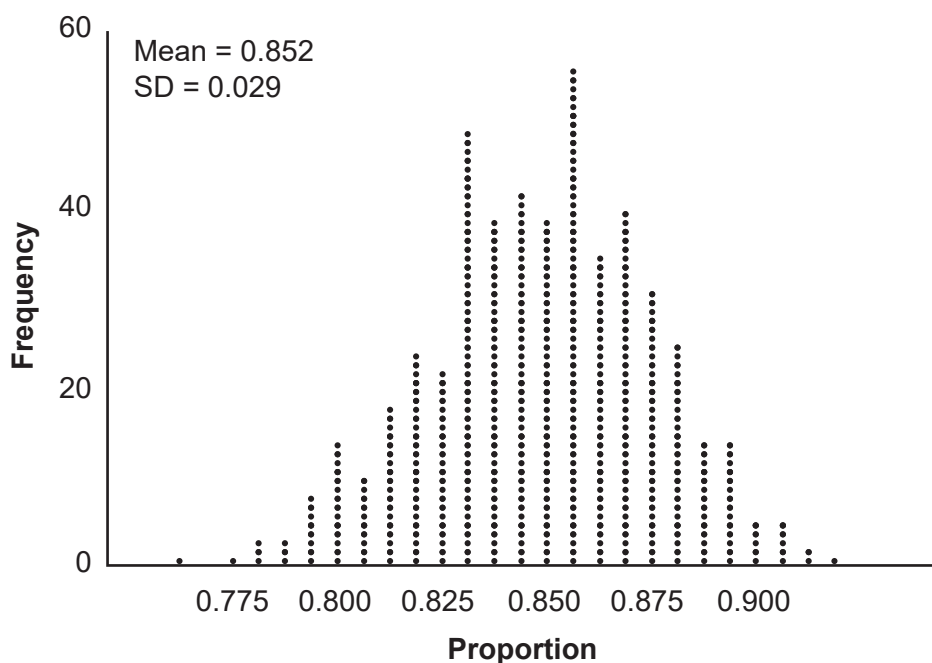
Suppose a sample of 150 students from your high school showed that 88% of students had purchased apps on their smartphones in the past 3 months. Based on the simulation, would the results from your high school give the app design company reason to believe their assumption is *incorrect*? Explain.

It falls within the 95% confidence interval of the simulations so the app design company has no reason to believe their assumption is incorrect.

**Score 1:** The student gave an incomplete explanation.

### Question 32

32 An app design company believes that the proportion of high school students who have purchased apps on their smartphones in the past 3 months is 0.85. A simulation of 500 samples of 150 students was run based on this proportion and the results are shown below.



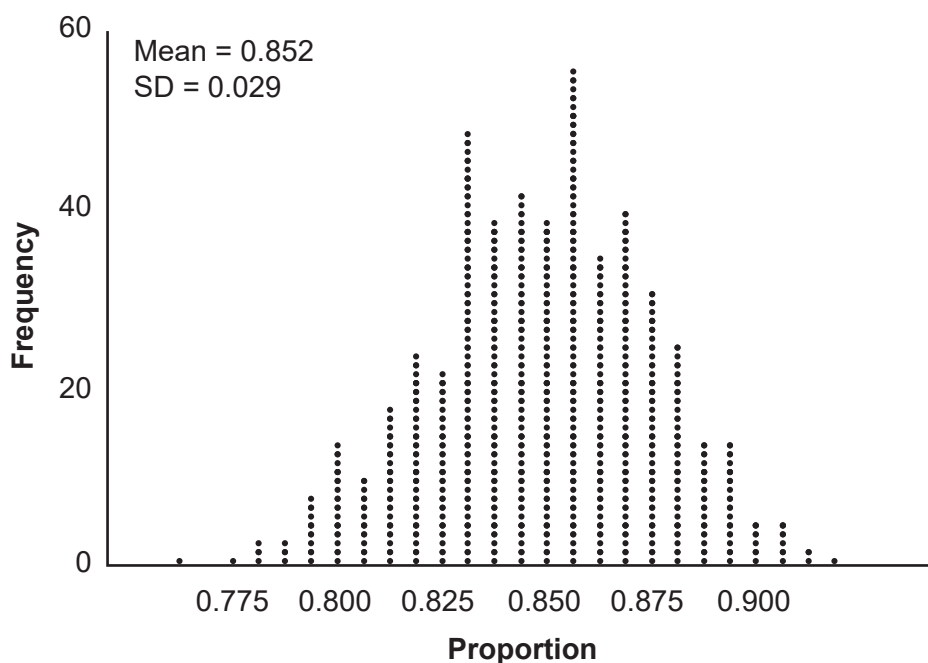
Suppose a sample of 150 students from your high school showed that 88% of students had purchased apps on their smartphones in the past 3 months. Based on the simulation, would the results from your high school give the app design company reason to believe their assumption is incorrect? Explain.

The only reason to believe this assumption is not correct due to the simulation is the fact that the highest/most likely proportion shown is 86% and not 88%.

**Score 0:** The student did not show enough relevant course-level work to receive any credit.

**Question 32**

- 32 An app design company believes that the proportion of high school students who have purchased apps on their smartphones in the past 3 months is 0.85. A simulation of 500 samples of 150 students was run based on this proportion and the results are shown below.



Suppose a sample of 150 students from your high school showed that 88% of students had purchased apps on their smartphones in the past 3 months. Based on the simulation, would the results from your high school give the app design company reason to believe their assumption is *incorrect*? Explain.

$\frac{96}{150} = 0.64$  Since the p value is 64%  
the data collected is not  
significant and, due to chance  
most likely  
so the app design company is  
incorrect.

**Score 0:** The student did not show enough relevant course-level work to receive any credit.

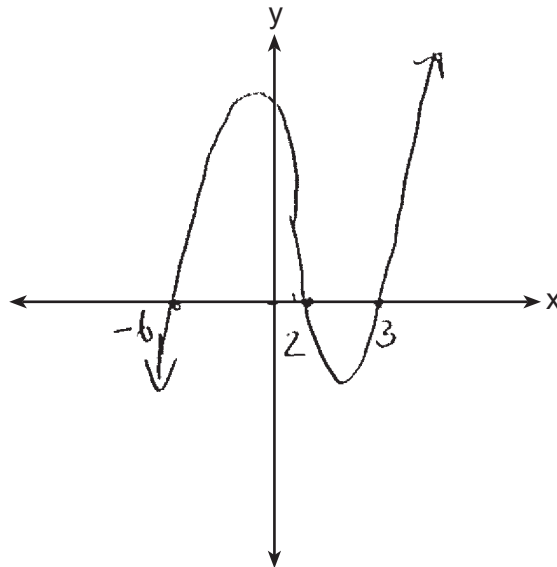
Question 33

33 Patricia creates a cubic polynomial function,  $p(x)$ , with a leading coefficient of 1. The zeros of the function are 2, 3, and  $-6$ . Write an equation for  $p(x)$ .

$$p(x) = (x-2)(x-3)(x+6)$$

$$\begin{array}{l} (x-2)(x-3) \\ x^2 - 3x - 2x + 6 \\ (x^2 - 5x + 6)(x+6) \\ x^3 - 5x^2 + 6x + 6x^2 + 36x + 36 \\ x^3 + x^2 + 41x + 36 \end{array}$$

Sketch  $y = p(x)$  on the set of axes below.



**Score 4:** The student gave a complete and correct response.



**Question 33**

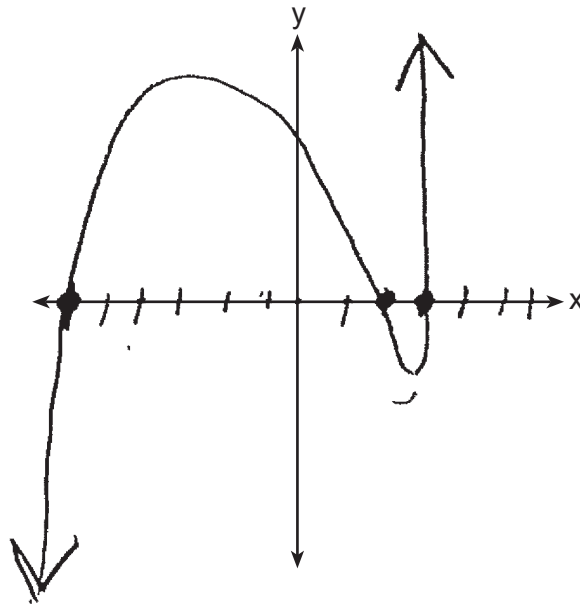
33 Patricia creates a cubic polynomial function,  $p(x)$ , with a leading coefficient of 1. The zeros of the function are 2, 3, and  $-6$ . Write an equation for  $p(x)$ .

$$p(x) = (x-2)(x-3)(x+6)$$
$$= x^2 - 5x + 6(x+6)$$

$$p(x) = x^3 + 6x^2 - 5x^2 - 30x + 6x + 36$$

$$p(x) = x^3 + x^2 - 24x + 36$$

Sketch  $y = p(x)$  on the set of axes below.



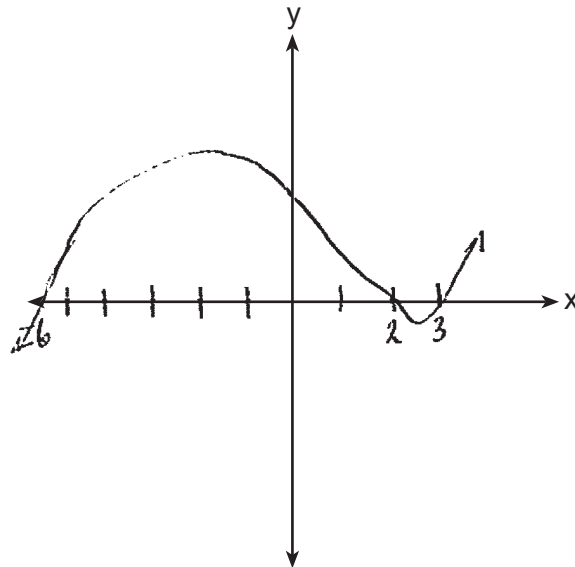
**Score 4:** The student gave a complete and correct response.

Question 33

33 Patricia creates a cubic polynomial function,  $p(x)$ , with a leading coefficient of 1. The zeros of the function are 2, 3, and  $-6$ . Write an equation for  $p(x)$ .

$$\begin{aligned} & (x-2)(x-3)(x-6) \\ & \quad x^2 - 3x - 2x + 6 \\ & (x-6)(x^2 - 5x + 6) \\ & \quad x^3 - 5x^2 + 6x - 6x^2 + 30x - 36 \\ & \boxed{p(x) = x^3 - 11x^2 + 36x - 36} \end{aligned}$$

Sketch  $y = p(x)$  on the set of axes below.



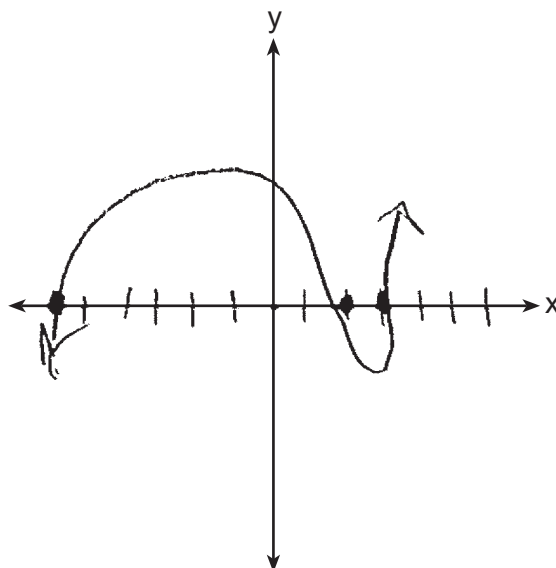
**Score 3:** The student incorrectly wrote one of the factors as  $x - 6$ .

**Question 33**

33 Patricia creates a cubic polynomial function,  $p(x)$ , with a leading coefficient of 1. The zeros of the function are 2, 3, and  $-6$ . Write an equation for  $p(x)$ .

$$(x-2)(x-3)(x+6)$$

Sketch  $y = p(x)$  on the set of axes below.



**Score 3:** The student wrote an expression, not an equation for  $p(x)$ . The student drew an acceptable sketch through the zeros with appropriate end behavior.

**Question 33**

33 Patricia creates a cubic polynomial function,  $p(x)$ , with a leading coefficient of 1. The zeros of the function are 2, 3, and  $-6$ . Write an equation for  $p(x)$ .

$$(x-2)(x-3)(x+6) = P(x)$$

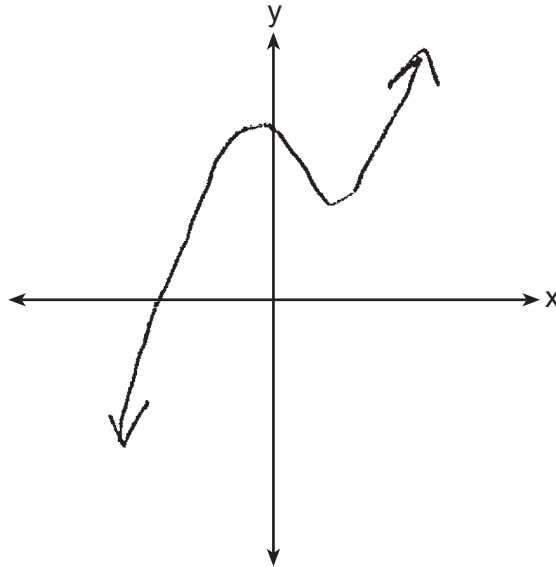
$$(x^2-3x-2x+6)(x+6) = P(x)$$

$$(x^2-5x+6)(x+6) = P(x)$$

$$x^3+6x^2-5x^2-30x+6x+36 = P(x)$$

$$x^3+x^2-24x+36 = P(x)$$

Sketch  $y = p(x)$  on the set of axes below.



**Score 2:** The student only received credit for the equation.

**Question 33**

**33** Patricia creates a cubic polynomial function,  $p(x)$ , with a leading coefficient of 1. The zeros of the function are 2, 3, and  $-6$ . Write an equation for  $p(x)$ .

$$x - (-6) \quad x - 2 \quad x - 3$$

$$(x - 2)(x - 3)(x + 6)$$

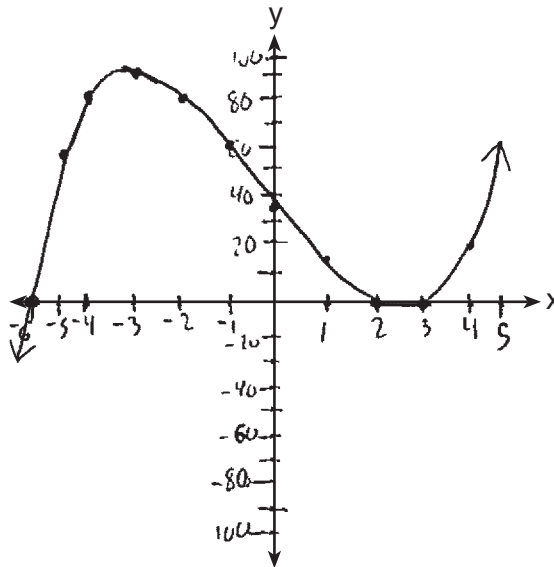
$$(x^2 - 3x - 2x + 6)(x + 6)$$

$$(x^2 - 5x + 6)(x + 6)$$

$$x^3 + 6x^2 - 5x^2 - 30x + 6x + 36$$

$$x^3 + x^2 - 24x + 36$$

Sketch  $y = p(x)$  on the set of axes below.



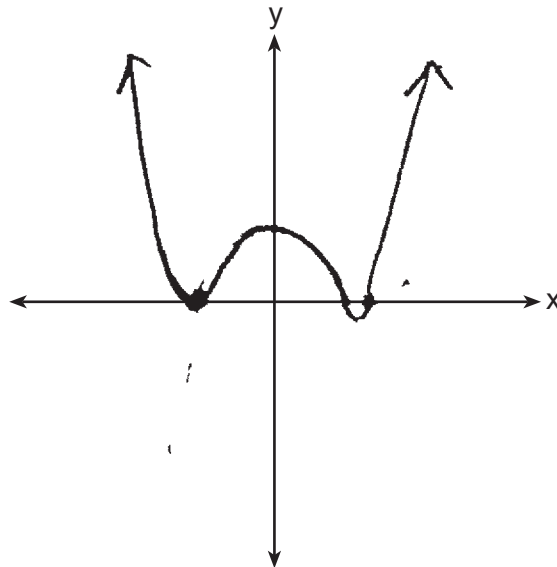
**Score 2:** The student did not write an equation for  $p(x)$  and made one graphing error.

Question 33

33 Patricia creates a cubic polynomial function,  $p(x)$ , with a leading coefficient of 1. The zeros of the function are 2, 3, and  $-6$ . Write an equation for  $p(x)$ .

$$P(x) = x^4 + 7x^3 - 18x^2 + 252x + 216$$
$$x^4 + 12x^3 + 36x^2 - 5x^3 - 60x^2 - 180x + 6x^3 + 7x^2$$
$$(x^2 - 5x + 6)(x^2 + 12x + 36)$$
$$(x-2)(x-3)(x+6)(x+6)$$

Sketch  $y = p(x)$  on the set of axes below.



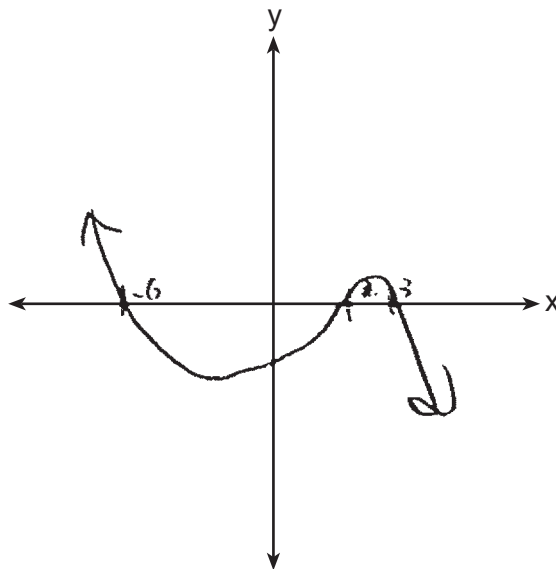
**Score 1:** The student received one credit for the sketch.

**Question 33**

**33** Patricia creates a cubic polynomial function,  $p(x)$ , with a leading coefficient of 1. The zeros of the function are 2, 3, and  $-6$ . Write an equation for  $p(x)$ .

$$P(x) =$$

Sketch  $y = p(x)$  on the set of axes below.



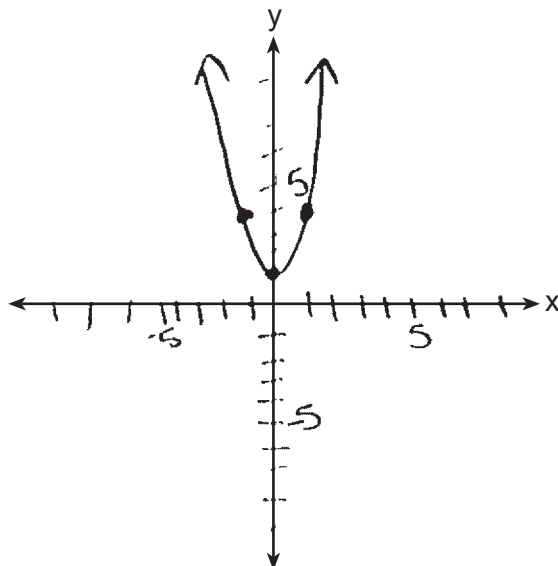
**Score 1:** The student received one credit for the sketch.

**Question 33**

33 Patricia creates a cubic polynomial function,  $p(x)$ , with a leading coefficient of 1. The zeros of the function are 2, 3, and  $-6$ . Write an equation for  $p(x)$ .

$$p(x) = 3x^2 + 1$$

Sketch  $y = p(x)$  on the set of axes below.



**Score 0:** The student did not show enough relevant course-level work to receive any credit.



Question 34

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

|                    |             | Donor Category |        |
|--------------------|-------------|----------------|--------|
|                    |             | Supporter      | Patron |
| Method of Donation | Phone calls | 400            | 672    |
|                    | Online      | 1200           | 2016   |

To the nearest thousandth find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$\begin{aligned}
 S &= \text{supporter} & P(S|O) &= \frac{1200}{3216} = \frac{25}{67} \approx \underline{.373} \text{ probability} \\
 O &= \text{online}
 \end{aligned}$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$P(S) = \frac{1600}{4286} = \frac{25}{67} \approx .373 \quad 37.3\%$$

Yes, because the probability of being a supporter is equal to the probability of being a supporter of those donating online.

**Score 4:** The student gave a complete and correct response.

**Question 34**

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

|                    |             | Donor Category |        |
|--------------------|-------------|----------------|--------|
|                    |             | Supporter      | Patron |
| Method of Donation | Phone calls | 400            | 672    |
|                    | Online      | 1200           | 2016   |
|                    |             | 1600           | 2688   |
|                    |             | = 3216         |        |
|                    |             | 1072           |        |
|                    |             | 4288           |        |

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$\frac{1200}{3216} = 0.373$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$P(O \cap S) \stackrel{?}{=} P(O) \cdot P(S)$$

$$\frac{1200}{4288} \stackrel{?}{=} \frac{3216}{4288} \cdot \frac{1600}{4288}$$

$$.2798507... \stackrel{?}{=} .2798507... \checkmark$$

Yes

**Score 4:** The student gave a complete and correct response.

**Question 34**

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

|                    |             | Donor Category |        |
|--------------------|-------------|----------------|--------|
|                    |             | Supporter      | Patron |
| Method of Donation | Phone calls | 400            | 672    |
|                    | Online      | 1200           | 2016   |

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$\frac{1200}{3216}$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$P(A|B) = P(A)$$

$$\frac{1200}{3216} = \frac{1600}{4288}$$

$$.3731343284 = .3731343284$$

yes being a supporter is independent of donating online

**Score 3:** The student did not round the conditional probability.

**Question 34**

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

|                    |             | Donor Category |        | T    |
|--------------------|-------------|----------------|--------|------|
|                    |             | Supporter      | Patron |      |
| Method of Donation | Phone calls | 400            | 672    | 1072 |
|                    | Online      | 1200           | 2016   | 3216 |
| T                  |             | 1600           | 2688   | 4288 |

To the nearest thousandth, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$P(S|O) = 1200 / 3216 = \frac{25}{67} = .373$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$P(S \cap O) = \frac{1200}{4288} = \frac{75}{268}$$

$$P(S) \cdot P(O) = \frac{3216}{4288} \cdot \frac{1600}{4288} = \frac{75}{268}$$

**Score 3:** The student did not indicate a positive response to indicate independence.

**Question 34**

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

|                    |             | Donor Category |        |
|--------------------|-------------|----------------|--------|
|                    |             | Supporter      | Patron |
| Method of Donation | Phone calls | 400            | 672    |
|                    | Online      | 1200           | 2016   |

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$P(S|O) = \frac{1200}{1600}$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$P(S) \cdot P(O) = P(S \cap O)$$

$$\frac{1600}{4288} \cdot \frac{3216}{4288} = \frac{1200}{4288}$$

$$.27985 = .27985$$

Yes b/c

$$P(S) \cdot P(O) = P(S \cap O)$$

**Score 2:** The student received no credit for the conditional probability.

**Question 34**

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

|                    |             | Donor Category |        |
|--------------------|-------------|----------------|--------|
|                    |             | Supporter      | Patron |
| Method of Donation | Phone calls | 400            | 672    |
|                    | Online      | 1200           | 2016   |

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$\frac{1200}{3216} = .3731 = .373$$

**.373**

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

No, as there are many more donors who are mostly donating online

**Score 2:** The student received no credit for determining independence.

**Question 34**

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

|                    |             | Donor Category |        | Total |
|--------------------|-------------|----------------|--------|-------|
|                    |             | Supporter      | Patron |       |
| Method of Donation | Phone calls | 400            | 672    | 1072  |
|                    | Online      | 1200           | 2016   | 3216  |
| Total              |             | 1600           | 2688   | 4288  |

To the nearest thousandth, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

bottom

$$P(S|O) = \frac{1600}{3216} = .4975$$

1200

.498

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$P(S+O) = P(S) \cdot P(O)$$

$$\frac{1200}{1600} = \frac{1600}{4288} \cdot \frac{3216}{4288}$$

.75  $\neq$  .2798507463

NOT independent

**Score 2:** The student made an error in the numerator of the conditional probability and an error calculating independence.

**Question 34**

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

|                    |             | Donor Category |        |
|--------------------|-------------|----------------|--------|
|                    |             | Supporter      | Patron |
| Method of Donation | Phone calls | 400            | 672    |
|                    | Online      | 1200           | 2016   |
|                    |             | 1600           | 2688   |

3216  
4241

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online. 510

$$\frac{1200}{3216}$$

$$37.31\%$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$P(A) \cdot P(B) = P(A) + P(B) - P(A \cup B)$$

$$(.7583)(.3772) = .28608$$

No they are not dependent because the possibility of  $P(A) \cdot P(B) \neq P(A) + P(B) - P(A \cup B)$

**Score 1:** The student received one credit for the exact conditional probability, but showed no further correct work.



**Question 34**

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

|                    |             | Donor Category |        |      |
|--------------------|-------------|----------------|--------|------|
|                    |             | Supporter      | Patron |      |
| Method of Donation | Phone calls | 400            | 672    | 1072 |
|                    | Online      | 1200           | 2016   | 3216 |
|                    |             | 1600           | 2688   | 4288 |

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$\frac{1200}{1600} = \boxed{.750}$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$\frac{400}{1600} = \boxed{.25} \quad \frac{1200}{1600} = \boxed{.75}$$

INDEPENDENT yes because different numbers

**Score 1:** The student found the conditional probability of the reversed conditions but showed no further correct work.

**Question 34**

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

|                    |             | Donor Category |        |
|--------------------|-------------|----------------|--------|
|                    |             | Supporter      | Patron |
| Method of Donation | Phone calls | 400            | 672    |
|                    | Online      | 1200           | 2016   |

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$\frac{1200}{4288} \quad \textcircled{.280}$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

Independent because  
there's multiple ways  
to support

**Score 0:** The student did not find a conditional probability and did not show enough relevant course-level work to receive any credit.

Question 35

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(x-2)^2 + (-2x+7-3)^2 = 20$$

$$x^2 - 4x + 4 + 4x^2 - 16x + 16 = 20$$

$$\frac{5x^2 - 20x + 20}{5} = \frac{20}{5}$$

$$x^2 - 4x + 4 = 4$$

$$\sqrt{(x-2)^2} = \sqrt{4} \quad y = -2x + 7$$

$$\begin{array}{l} x-2 = \pm 2 \\ \quad +2 \quad +2 \\ x = 2 \pm 2 \\ x = 0, \quad x = 4 \end{array} \quad \begin{array}{l} y = -2(0) + 7 \\ y = 0 + 7 \\ y = 7 \end{array} \quad \begin{array}{l} y = -2(4) + 7 \\ y = -8 + 7 \\ y = -1 \end{array}$$

$$\boxed{(0, 7) \quad (4, -1)}$$

**Score 4:** The student gave a complete and correct response.

Question 35

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(x-2)(x-2) + (y-3)(y-3) = 20$$

$$x^2 - 4x + 4 + y^2 - 6y + 9 = \cancel{20}$$

$$\phantom{x^2 - 4x + 4 + y^2 - 6y + 9 = \cancel{20}} - 20 \phantom{x^2 - 4x + 4 + y^2 - 6y + 9 = \cancel{20}} - 20$$

$$x^2 - 4x + y^2 - 6y - 7 = 0$$

$$x^2 - 4x + (-2x+7)(-2x+7) - 6(-2x+7) - 7 = 0$$

~~$$x^2 - 4x + 4x^2 - 28x + 49 + 12x - 42 - 7 = 0$$~~

$$5x^2 - 20x = 0$$

$$5x(x - 4) = 0$$

$$5x = 0$$

$$\frac{\quad}{5}$$

$$x = 0$$

~~$$x - 4 = 0$$~~

~~$$+4 +4$$~~

~~$$\hline$$~~

~~$$x = 4$$~~

$$y = -2(0) + 7 \quad y = -2(4) + 7$$

$$y = 7$$

$$y = -1$$

**Score 4:** The student gave a complete and correct response.

**Question 35**

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(x-2)^2 + (-2x+7-3)^2 = 20 \rightarrow (x-2)^2 + (-2x+4)^2$$

$$x^2 - 4x + 4 + 4x^2 - 16x + 16 = 20$$

$$5x^2 - 20x + 20 = 20$$

$$-20 \quad -20$$

$$5x^2 - 20x = 0$$

$$5x(x-4)$$

$$x = 4$$

$$y = -1$$

**Score 3:** The student found only one solution.

Question 35

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(x-2)^2 + (-2x+7-3)^2 = 20$$

$$x^2 - 4x + 4 + 4x^2 - 16x + 16 = 20$$

$$5x^2 - 20x + 20 = 20$$

$$5x^2 - 20x = 0$$

$$5x(x-4) = 0$$

|       |       |
|-------|-------|
| $x=0$ | $x=4$ |
|-------|-------|

$$(0-2)^2 + (y-3)^2 = 20$$

$$4 + y^2 - 6y + 9 = 20$$

$$y^2 - 6y + 13 = 20$$

$$y^2 - 6y - 7 = 0$$

$$y^2 - y + 7y - 7$$

$$y(y-1) + 7(y-1)$$

$$(y-1)(y+7)$$

|       |
|-------|
| $y=1$ |
|-------|

|        |
|--------|
| $y=-7$ |
|--------|

Score 3: The student received credit for both x-values.

Question 35

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$\begin{array}{r} x-2 \\ x \begin{array}{|l} x^2-2x \\ \hline 2x-4 \end{array} \\ -2 \end{array}$$

$$\begin{array}{r} x-3 \\ x \begin{array}{|l} x^2-3x \\ \hline -3x+9 \end{array} \\ -3 \end{array}$$

$$x^2 - 4x + 4 + y^2 - 6x + 9 = 20$$

$$\underline{\hspace{10em}} \quad 5x - 14 = 20$$

$$\begin{array}{r} x-4=0 \\ +4=4 \\ \hline x=4 \end{array}$$

$$(-2x + 7 - 3)^2 = 20$$

$$(-2x + 4)^2 = 20$$

$$\begin{array}{r} -2x+4 \\ -2x \begin{array}{|l} 4x^2-8x \\ \hline -8x+16 \end{array} \\ +4 \end{array}$$

$$x=4$$

$$x^2 - 4x + 4 + 4x^2 - 16x + 16$$

$$5x^2 - 20x + 20 = 20$$

$$\underline{\hspace{10em}} \quad 5x^2 - 20x = 0$$

$$5(x-4) = 0$$

**Score 2:** The student wrote a correct quadratic in standard form.

Question 35

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(y-3)^2 = 20 - (x-2)^2$$

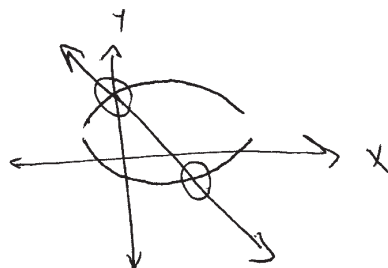
$$y-3 = \pm \sqrt{20 - (x-2)^2}$$

$$y = \pm \sqrt{20 - (x-2)^2} + 3$$

$$Y_1 = \sqrt{20 - (x-2)^2} + 3$$

$$Y_2 = -\sqrt{20 - (x-2)^2} + 3$$

$$Y_3 = -2x + 7$$



| x | y <sub>1</sub> | y <sub>2</sub> | y <sub>3</sub> |
|---|----------------|----------------|----------------|
| 0 | 7              | -1             | 7              |
| 1 |                |                |                |
| 2 |                |                |                |
| 3 |                |                |                |
| 4 | 7              | -1             | -1             |

(0, 7)  
and  
(4, -1)

**Score 2:** The student solved correctly but used a method other than algebraic.



Question 35

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(x-2)(x-2) + (-2x+7-3)(-2x+7-3) = 20$$

$$(x^2 - 4x + 4) + (4x^2 - 14x + 6x - 14x + 49 - 2)$$

$$+ 6x - 21 + 9$$
~~$$(x^2 - 4x + 4) + (4x^2 - 22x - 28 + 6x - 21 + 4)$$~~

$$(x^2 - 4x + 4) + (4x^2 - 16x - 40)$$

$$5x^2 - 20x - 36 = 20$$

$$\begin{array}{r} 5x^2 - 20x - 36 = 20 \\ \hline 5x^2 - 20x - 56 = 6 \\ \hline \end{array}$$

A      B      C

**Score 1:** The student made one computational error when attempting to put the equation in standard form, but showed no further correct work.

**Question 35**

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(x-2)(x-2) + (y-3)(y-3) = 20$$

$$(x-2)^2 + (-2x+7-3)^2 = 20$$

$$x^2 - 4 + -4x^2 + 14 - 6 = 20$$

$$\begin{array}{r} -4x^2 + 4 = 20 \\ -4 \quad -4 \end{array}$$

$$\begin{array}{r} -4x^2 = 16 \\ -4 \quad -4 \end{array}$$

$$\sqrt{x^2} = \sqrt{-4}$$

$$x = 2i$$

**Score 0:** The student did not show enough correct course-level work to receive any credit.

Question 35

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(x - 2)^2 + (-2x + 7 - 3)^2 = 20$$

$$x - 4 + 2x + 7 - 6 = 20$$

$$\begin{array}{r} 3x - 3 = 20 \\ +3 \quad +3 \\ \hline \end{array}$$

$$\begin{array}{r} 3x = 23 \\ \hline 3 \quad 3 \end{array}$$

$$\boxed{x = 8}$$

$$(8, -9)$$

$$y = -2(8) + 7$$

$$y = -16 + 7$$

$$\boxed{y = -9}$$

**Score 0:** The student did not show enough course-level work to receive any credit.

**Question 36**

**36** On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

Write two functions,  $P(x)$  and  $F(x)$ , that represent the number of palm trees and flamingos on this island, respectively,  $x$  years from now.

$$P(x) = 500 (.97)^x$$
$$F(x) = 200e^{.02x}$$

State the solution to the equation  $P(x) = F(x)$ , rounded to the *nearest year*. Interpret the meaning of this value within the given context.

$$x = 18$$

In 18 years the number of flamingos and palm trees will be equal.

**Score 4:** The student gave a complete and correct response.

**Question 36**

**36** On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

Write two functions,  $P(x)$  and  $F(x)$ , that represent the number of palm trees and flamingos on this island, respectively,  $x$  years from now.

$$P(x) = 500 (1 - .03)^x$$

$$F(x) = 200 e^{.02(x)}$$

State the solution to the equation  $P(x) = F(x)$ , rounded to the *nearest year*. Interpret the meaning of this value within the given context.

When they have the exact same amount of flamingos and palm trees, it will be in 18 years.

**Score 4:** The student gave a complete and correct response.

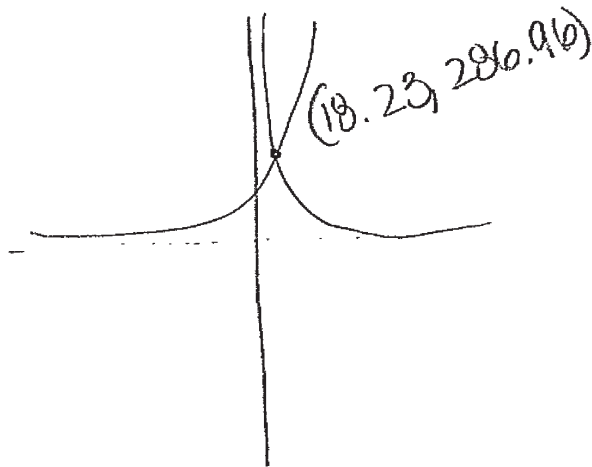
**Question 36**

**36** On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

Write two functions,  $P(x)$  and  $F(x)$ , that represent the number of palm trees and flamingos on this island, respectively,  $x$  years from now.

$$P(x) = 500(1 - 0.03)^x$$
$$F(x) = 200(1 + 0.02)^x$$

State the solution to the equation  $P(x) = F(x)$ , rounded to the *nearest year*. Interpret the meaning of this value within the given context.



In 18 years the palm tree and flamingo population will be equal.

**Score 3:** The student created an incorrect equation for  $F(x)$ .

**Question 36**

**36** On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

$$y = a(b)^x$$

$$A = Pe^{rt}$$

Write two functions,  $P(x)$  and  $F(x)$ , that represent the number of palm trees and flamingos on this island, respectively,  $x$  years from now.

$$P_x = 500(.97)^x$$

$$F_x = 200e^{.02x}$$

State the solution to the equation  $P(x) = F(x)$ , rounded to the *nearest year*. Interpret the meaning of this value within the given context.

| $x$ | $Y_1$  | $Y_2$  |
|-----|--------|--------|
| 15  | 316.63 | 269.97 |
| 16  | 307.13 | 275.43 |
| 17  | 297.91 | 280.99 |
| 18  | 288.98 | 286.98 |
| 19  | 280.31 | 292.46 |

18 years

**Score 3:** The student did not interpret the meaning of 18 years.

**Question 36**

36 On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

Write two functions,  $P(x)$  and  $F(x)$ , that represent the number of palm trees and flamingos on this island, respectively,  $x$  years from now.

$$P(x) = 500e^{-.03x}$$

$$F(x) = 200e^{.02x}$$

State the solution to the equation  $P(x) = F(x)$ , rounded to the *nearest year*. Interpret the meaning of this value within the given context.

The solution to the equation is 18.3, which means that at 18.3 years, the populations will have the same amount, and then the tree population will continue decreasing and the flamingo population will continue increasing.

**Score 2:** The student wrote an incorrect equation for  $P(x)$  and rounded the solution to  $P(x) = F(x)$  incorrectly.



Question 36

36 On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

Write two functions,  $P(x)$  and  $F(x)$ , that represent the number of palm trees and flamingos on this island, respectively,  $x$  years from now.

$$P(x) = 500(0.97)^x$$
$$f(x) = 200(1.02)^x$$

State the solution to the equation  $P(x) = F(x)$ , rounded to the nearest year. Interpret the meaning of this value within the given context.

$$P(x) = f(x)$$

$$\frac{500(0.97)^x}{200} = \frac{200(1.02)^x}{200}$$

$$2.5(0.97)^x = (1.02)^x$$

$$x \frac{\log(2.5)}{\log 1.02} = x \frac{\log 1.02}{\log 1.02}$$

$$x = 44.73$$

**Score 1:** The student received one credit for correctly writing  $P(x)$ .

**Question 36**

**36** On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

Write two functions,  $P(x)$  and  $F(x)$ , that represent the number of palm trees and flamingos on this island, respectively,  $x$  years from now.

$$A = P(1+r)^t \quad \text{palm trees} \quad \frac{x}{365}$$
$$A = 500(1 - 0.03)^{\frac{x}{365}}$$

flamingos

$$A = 500e^{(0.02)\frac{x}{365}}$$

Part

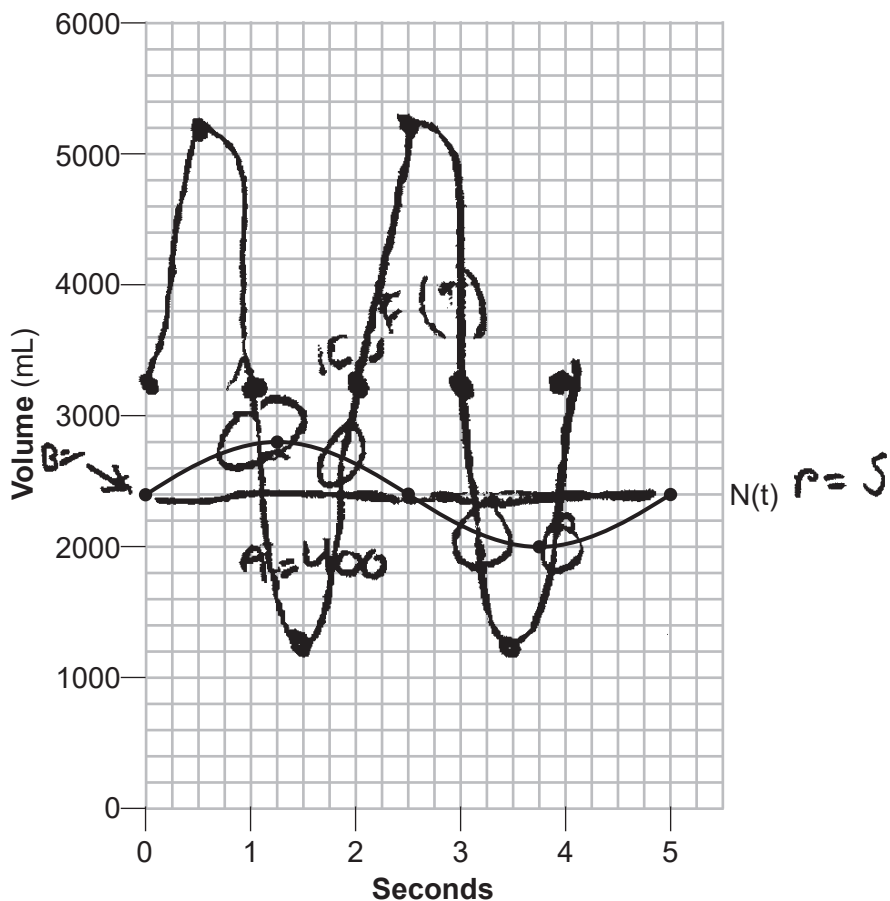
State the solution to the equation  $P(x) = F(x)$ , rounded to the *nearest year*. Interpret the meaning of this value within the given context.

The  $x$ -coordinate of the solution to equation  $P(x) = F(x)$  is at 0. The meaning of the value in this context is that it is the point where they intersect, thus they are the same.

**Score 0:** The student did not show enough correct work to receive any credit.

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for  $N(t)$ , in the form  $N(t) = A \sin(Bt) + C$ .

Handwritten work:

$$A = 400$$

$$B = \frac{2\pi}{P} = \frac{2\pi}{3}$$

$$C = 2400$$

$$N(t) = 400 \sin\left(\frac{2\pi}{3}t\right) + C$$

Question 37 is continued on the next page.

**Score 6:** The student gave a complete and correct response.

---

**Question 37**

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**Question 37 continued**

That same lung, when engaged in exercise, has a volume that can be modeled by  $E(t) = 2000 \sin(\pi t) + 3200$ , where  $E(t)$  is volume in mL and  $t$  is time in seconds.

Graph *at least one* cycle of  $E(t)$  on the same grid as  $N(t)$ .

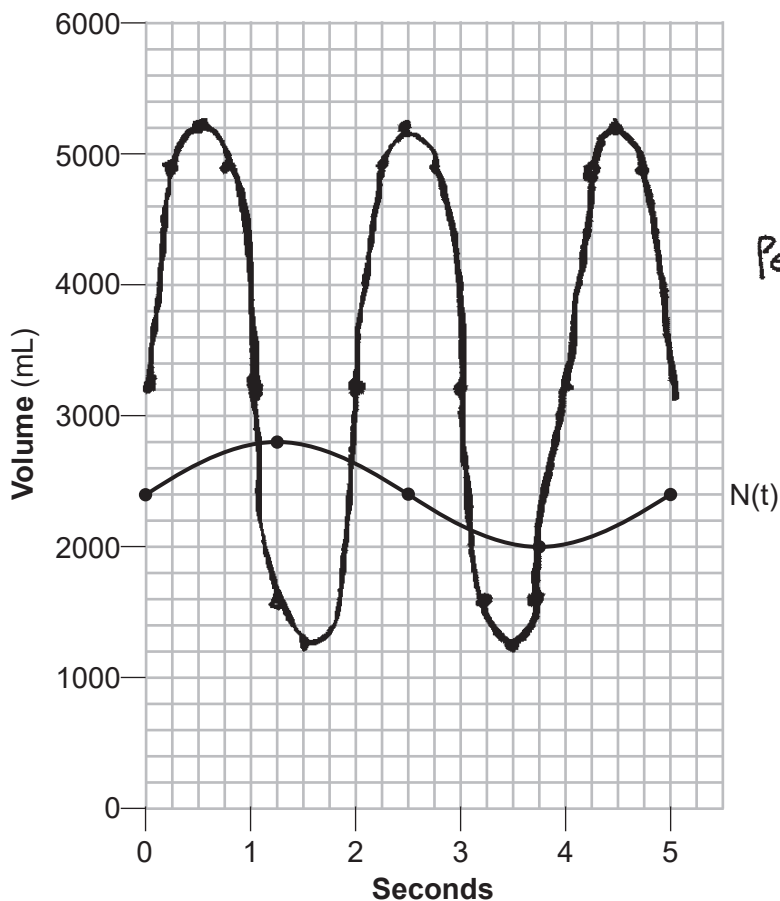
$$P = \frac{B}{8} = 2 \quad B = 3200$$
$$A = 2000 \quad :$$

How many times during the 5-second interval will  $N(t) = E(t)$ ?

4

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for  $N(t)$ , in the form  $N(t) = A \sin(Bt) + C$ .

$$N(t) = 400 \sin\left(\frac{2\pi}{5}t\right) + 2400$$

Question 37 is continued on the next page.

**Score 6:** The student gave a complete and correct response.

---

**Question 37**

---

**Question 37 continued**

That same lung, when engaged in exercise, has a volume that can be modeled by  $E(t) = 2000 \sin(\pi t) + 3200$ , where  $E(t)$  is volume in mL and  $t$  is time in seconds.

Graph *at least one* cycle of  $E(t)$  on the same grid as  $N(t)$ .

$$x = \frac{2\pi}{\pi}$$

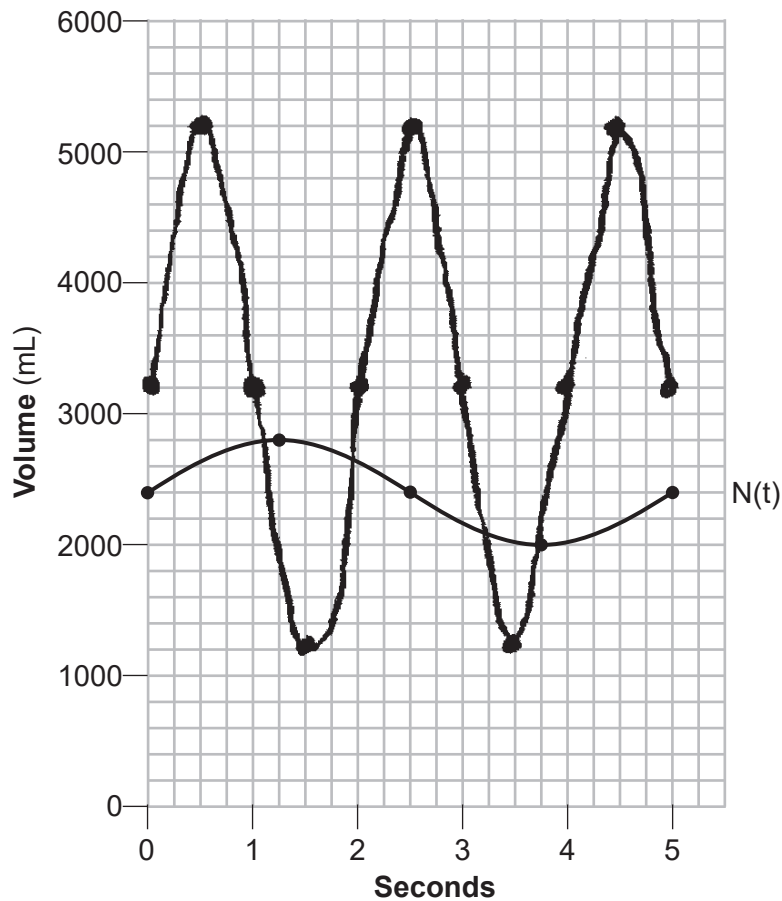
$$x = 2$$

How many times during the 5-second interval will  $N(t) = E(t)$ ?

4

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for  $N(t)$ , in the form  $N(t) = A \sin(Bt) + C$ .

$$N(t) = 400 \sin\left(\frac{\pi}{4} t\right) + 2400$$

Question 37 is continued on the next page.

**Score 5:** The student incorrectly found the value for  $B$ .

---

**Question 37**

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**Question 37 continued**

That same lung, when engaged in exercise, has a volume that can be modeled by  $E(t) = 2000 \sin(\pi t) + 3200$ , where  $E(t)$  is volume in mL and  $t$  is time in seconds.

Graph *at least one* cycle of  $E(t)$  on the same grid as  $N(t)$ .

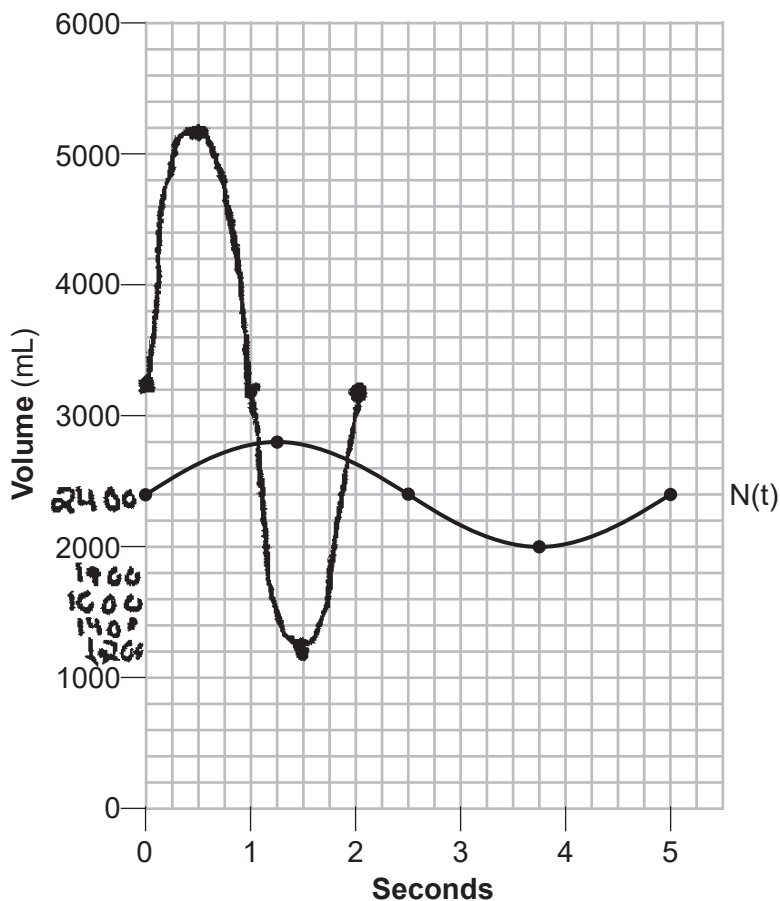
How many times during the 5-second interval will  $N(t) = E(t)$ ?

4



Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for  $N(t)$ , in the form  $N(t) = A \sin(Bt) + C$ .

$$\begin{aligned}
 Bp &= 2\pi \\
 B(5) &= 2\pi \\
 B &= \frac{2\pi}{5}
 \end{aligned}$$

$$\begin{aligned}
 C &= 2400 \\
 A &= 400
 \end{aligned}$$

$$N(t) = 400 \sin\left(\frac{2\pi}{5}t\right) + 2400$$

Question 37 is continued on the next page.

**Score 5:** The student did not find all the intersections within the 5 second interval.

---

**Question 37**

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**Question 37 continued**

That same lung, when engaged in exercise, has a volume that can be modeled by  $E(t) = 2000 \sin(\pi t) + 3200$ , where  $E(t)$  is volume in mL and  $t$  is time in seconds.

Graph *at least one* cycle of  $E(t)$  on the same grid as  $N(t)$ .

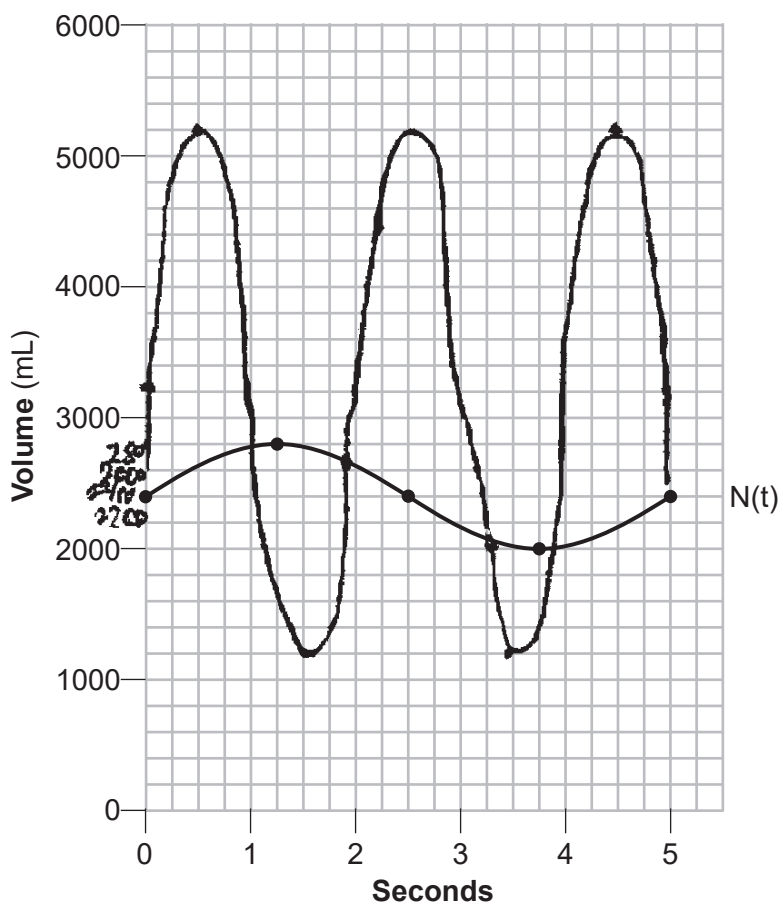
$$\begin{array}{l} Bp = 2\pi \\ \pi p = 2\pi \end{array} \quad p = 2 \text{ sec}$$

How many times during the 5-second interval will  $N(t) = E(t)$ ?

2

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for  $N(t)$ , in the form  $N(t) = A \sin(Bt) + C$ .

$$\frac{2800 - 2000}{2} = 400$$

amp  
period = 5

$$\frac{2\pi}{5} = 5$$

$$\frac{2\pi}{5} = \frac{5B}{5}$$

$$B = \frac{2\pi}{5} \text{ midline} = 2400$$

$$N(t) = 400 \sin(Bt) + C$$

$$N(t) = 400 \sin\left(\frac{2\pi}{5}t\right) + C$$

$$N(t) = 400 \sin\left(\frac{2\pi}{5}t\right) + 2400$$

Question 37 is continued on the next page.

Score 5: The student made a notation error.

---

**Question 37**

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**Question 37 continued**

That same lung, when engaged in exercise, has a volume that can be modeled by  $E(t) = 2000 \sin(\pi t) + 3200$ , where  $E(t)$  is volume in mL and  $t$  is time in seconds.

Graph *at least one cycle* of  $E(t)$  on the same grid as  $N(t)$ .

period =  $\frac{2\pi}{\pi} = 2$   
amplitude: 2000 mid line = 3200

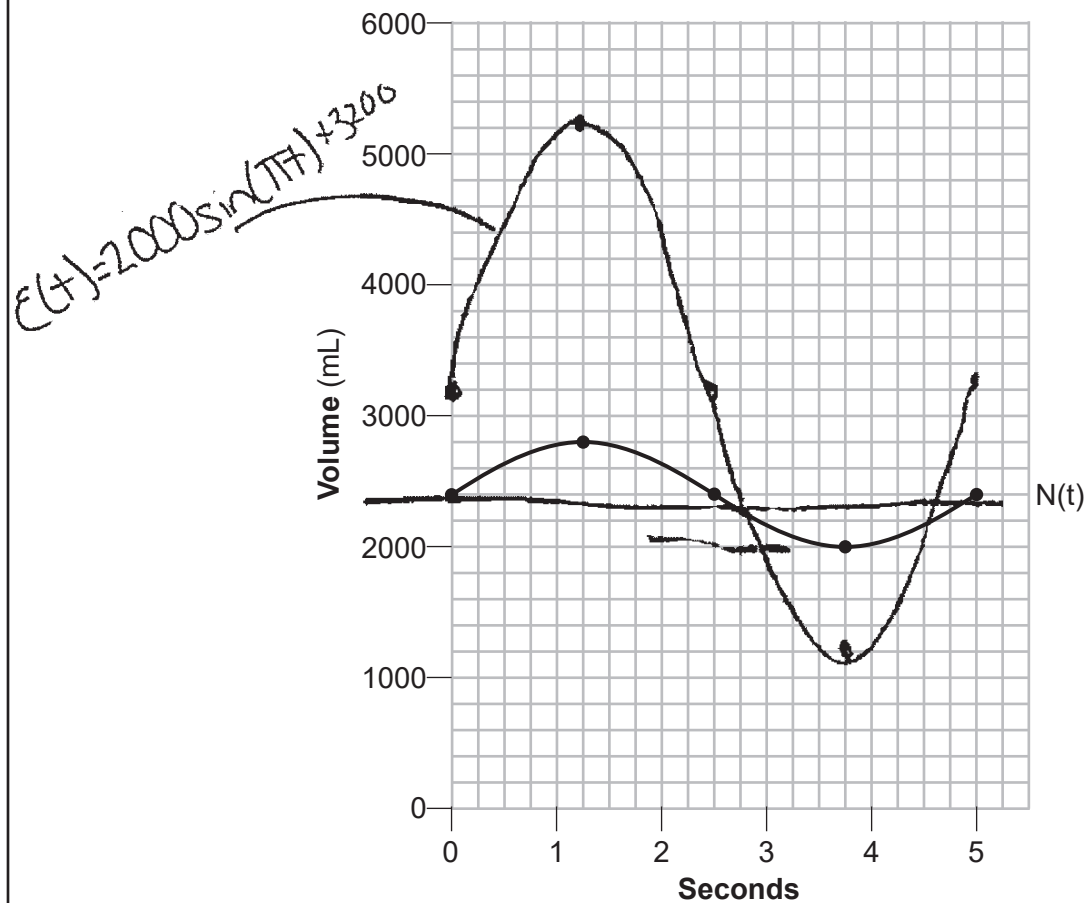
| x             | y    |
|---------------|------|
| 0             | 3200 |
| $\frac{1}{2}$ | 5200 |
| 1             | 3200 |
| $\frac{3}{2}$ | 1200 |
| 2             | 3200 |
| $\frac{5}{2}$ | 5200 |

How many times during the 5-second interval will  $N(t) = E(t)$ ?

4

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for  $N(t)$ , in the form  $N(t) = A \sin(Bt) + C$ .

$$N(t) = 4000 \sin(\pi t) + 2400$$

Question 37 is continued on the next page.

**Score 4:** The student did not find the correct value for  $B$  and did not graph the correct period.

---

**Question 37**

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**Question 37 continued**

That same lung, when engaged in exercise, has a volume that can be modeled by  $E(t) = 2000 \sin(\pi t) + 3200$ , where  $E(t)$  is volume in mL and  $t$  is time in seconds.

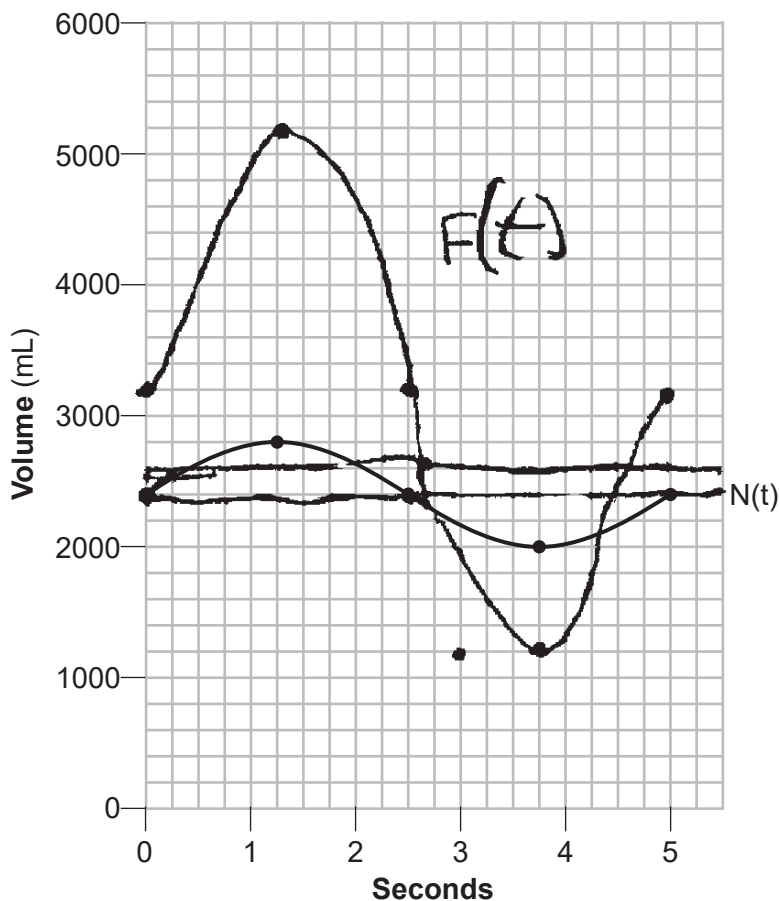
Graph *at least one* cycle of  $E(t)$  on the same grid as  $N(t)$ .

How many times during the 5-second interval will  $N(t) = E(t)$ ?

2

**Question 37**

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for  $N(t)$ , in the form  $N(t) = A \sin(Bt) + C$ .

$$N(t) = A \sin(Bt) + C$$

$$\frac{5}{1} = \frac{2\pi}{b}$$

$$\frac{5}{5} = \frac{2\pi}{5}$$

$$4100 \sin\left(\frac{2\pi}{5}t\right) + 2100$$

**Question 37 is continued on the next page.**

**Score 4:** The student did not write an equation and did not graph the correct period for  $E(t)$ .

---

**Question 37**

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**Question 37 continued**

That same lung, when engaged in exercise, has a volume that can be modeled by  $E(t) = 2000 \sin(\pi t) + 3200$ , where  $E(t)$  is volume in mL and  $t$  is time in seconds.

Graph *at least one* cycle of  $E(t)$  on the same grid as  $N(t)$ .

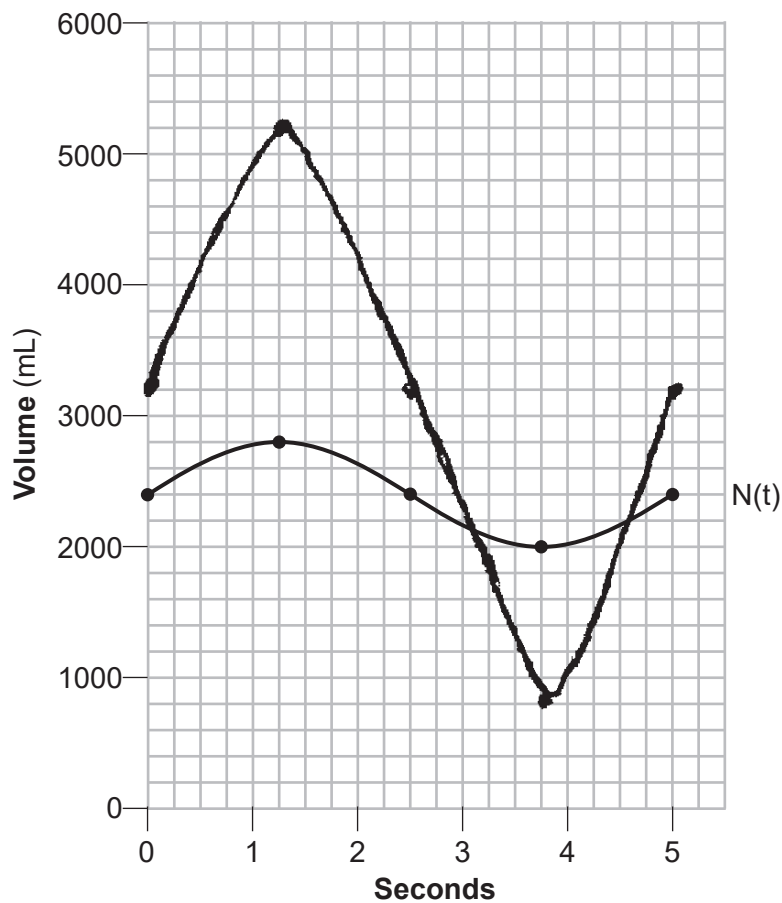
How many times during the 5-second interval will  $N(t) = E(t)$ ?

2



Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for  $N(t)$ , in the form  $N(t) = A \sin(Bt) + C$ .

In five seconds  $\rightarrow$   $N(t) = 400 \sin(\pi t) + 2400$

Question 37 is continued on the next page.

**Score 3:** The student did not find the correct value for  $B$ , and made two graphing errors.

---

**Question 37**

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**Question 37 continued**

That same lung, when engaged in exercise, has a volume that can be modeled by  $E(t) = 2000 \sin(\pi t) + 3200$ , where  $E(t)$  is volume in mL and  $t$  is time in seconds.

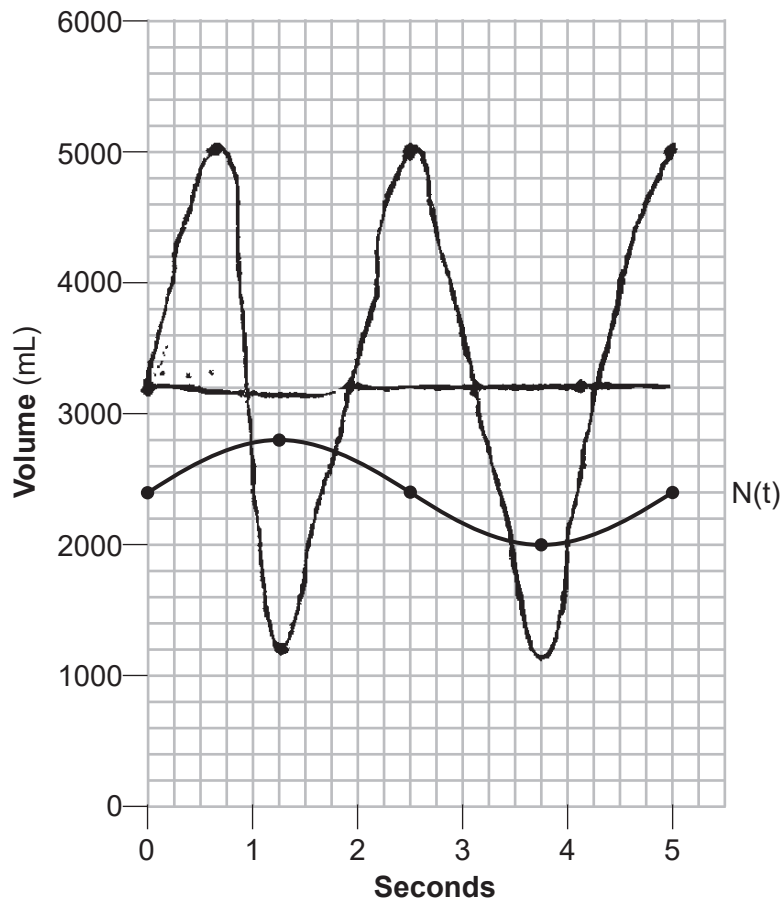
Graph *at least one* cycle of  $E(t)$  on the same grid as  $N(t)$ .

How many times during the 5-second interval will  $N(t) = E(t)$ ?

~~two~~ two locations.

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for  $N(t)$ , in the form  $N(t) = A \sin(Bt) + C$ .

$$N(t) = 400 \sin\left(\frac{2\pi}{5}x\right) + 2400$$

Question 37 is continued on the next page.

**Score 3:** The student did not graph the correct amplitude or period and made a notation error.

---

**Question 37**

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**Question 37 continued**

That same lung, when engaged in exercise, has a volume that can be modeled by  $E(t) = 2000 \sin(\pi t) + 3200$ , where  $E(t)$  is volume in mL and  $t$  is time in seconds.

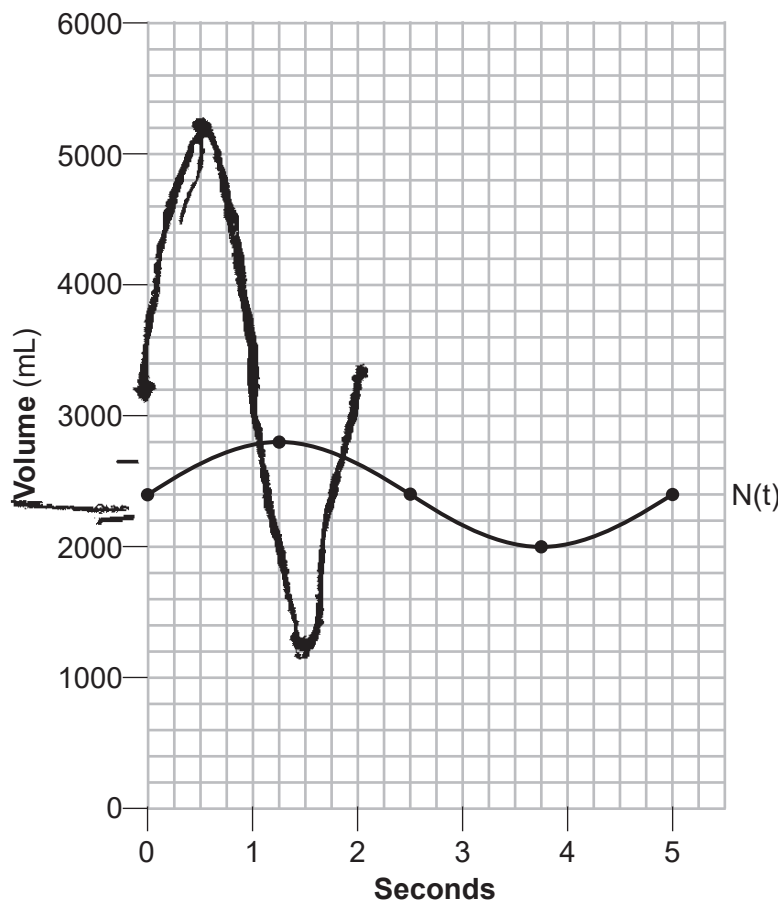
Graph *at least one* cycle of  $E(t)$  on the same grid as  $N(t)$ .

How many times during the 5-second interval will  $N(t) = E(t)$ ?

4

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



$$a = 400$$

$$b = \frac{5}{2\pi}$$

$$C = 2200$$

$$p = 5$$

$$5 = \frac{2\pi}{b}$$

$$\frac{5}{2\pi} = b$$

Using the graph, write an equation for  $N(t)$ , in the form  $N(t) = A \sin(Bt) + C$ .

$$N(t) = 400 \sin\left(\frac{5}{2\pi} t\right) + 2200$$

Question 37 is continued on the next page.

**Score 3:** The student did not find the correct values of  $B$  and  $C$  for  $N(t)$  and did not graph the correct midline for  $E(t)$ .

---

**Question 37**

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**Question 37 continued**

That same lung, when engaged in exercise, has a volume that can be modeled by  $E(t) = 2000 \sin(\pi t) + 3200$ , where  $E(t)$  is volume in mL and  $t$  is time in seconds.

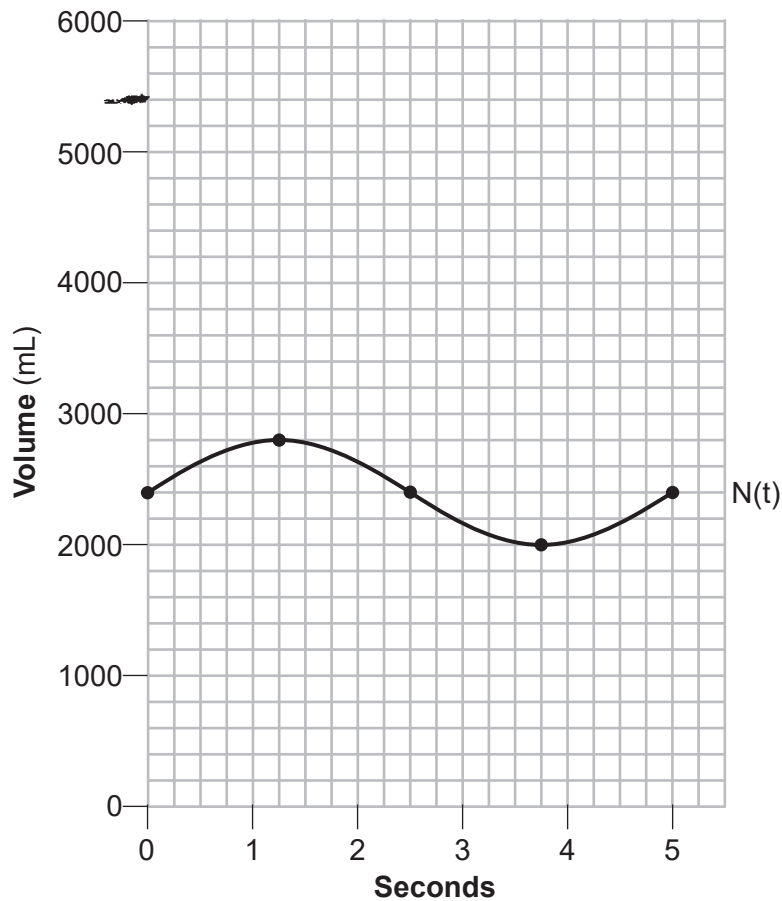
Graph *at least one* cycle of  $E(t)$  on the same grid as  $N(t)$ .

How many times during the 5-second interval will  $N(t) = E(t)$ ?

4 locations

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for  $N(t)$ , in the form  $N(t) = A \sin(Bt) + C$ .

$$\begin{aligned} A &= 400 \\ B &= \frac{2\pi}{5} \\ C &= 2200 \end{aligned}$$

$$N(t) = 400 \sin\left(\frac{2\pi}{5}t\right) + 2200$$

Question 37 is continued on the next page.

**Score 2:** The student earned one point for the  $N(t)$  equation and one point for the number of intersections.

---

**Question 37**

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**Question 37 continued**

That same lung, when engaged in exercise, has a volume that can be modeled by  $E(t) = 2000 \sin(\pi t) + 3200$ , where  $E(t)$  is volume in mL and  $t$  is time in seconds.

Graph *at least one* cycle of  $E(t)$  on the same grid as  $N(t)$ .

$$p = \frac{2\pi}{T} \quad 2$$

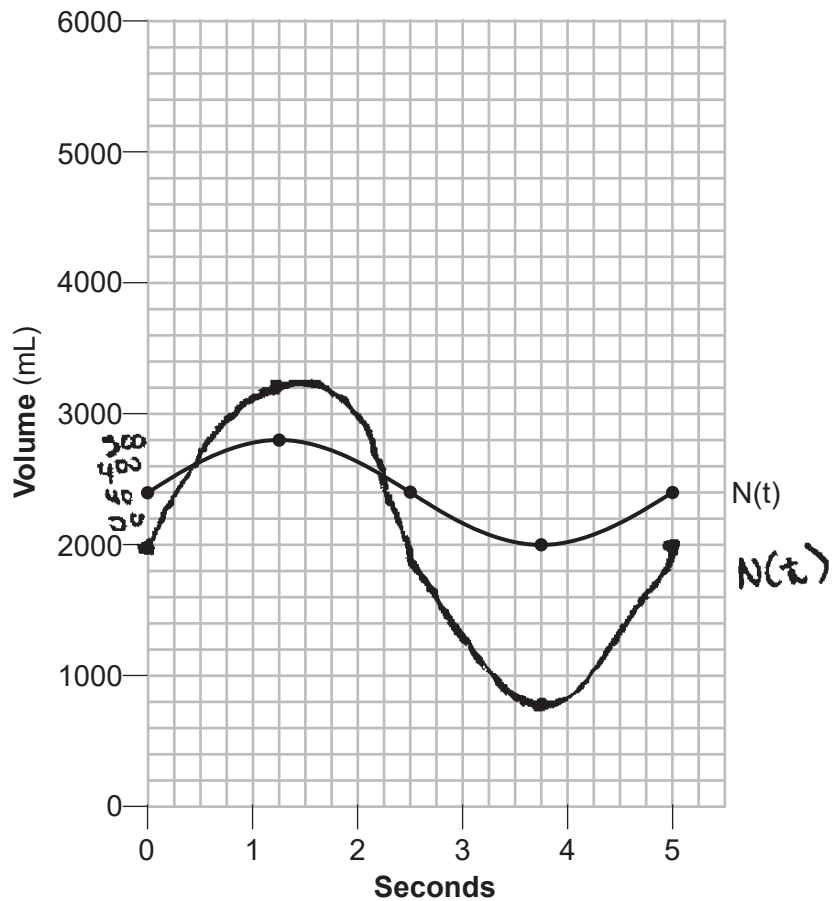
How many times during the 5-second interval will  $N(t) = E(t)$ ?

4 times



Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for  $N(t)$ , in the form  $N(t) = A \sin(Bt) + C$ .

$$N(t) = 2000 \sin(\pi t) + 2800$$

Question 37 is continued on the next page.

**Score 1:** The student found the correct number of intersections based on their graph.

---

**Question 37**

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**Question 37 continued**

That same lung, when engaged in exercise, has a volume that can be modeled by  $E(t) = 2000 \sin(\pi t) + 3200$ , where  $E(t)$  is volume in mL and  $t$  is time in seconds.

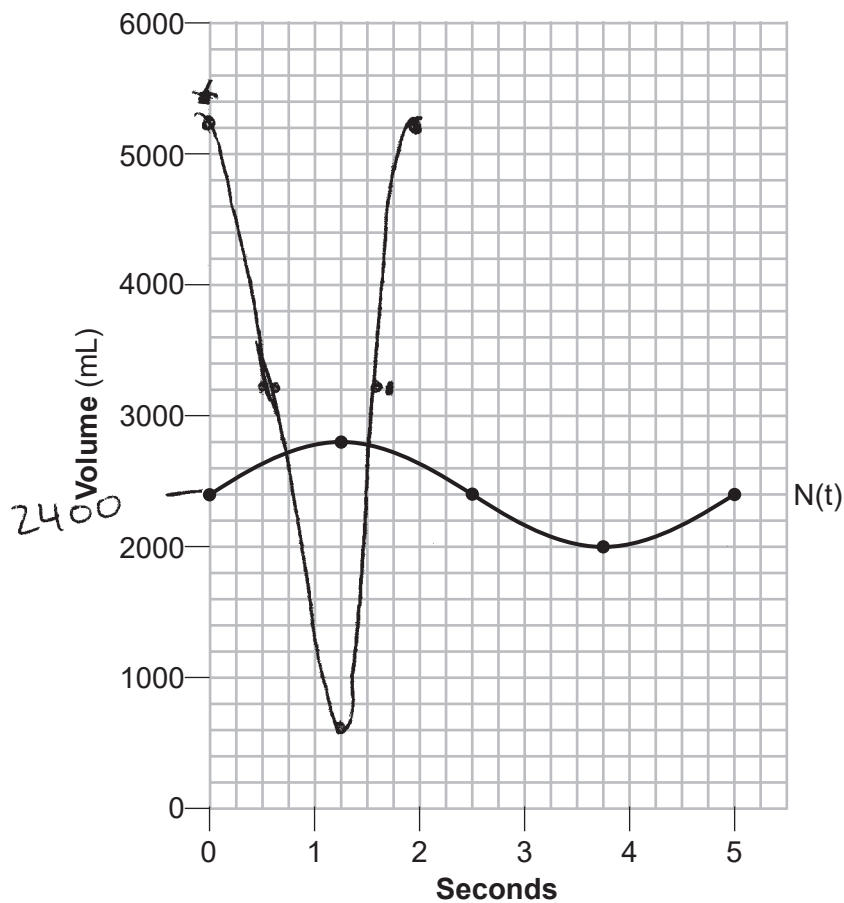
Graph *at least one* cycle of  $E(t)$  on the same grid as  $N(t)$ .

How many times during the 5-second interval will  $N(t) = E(t)$ ?

Two (2)

**Question 37**

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for  $N(t)$ , in the form  $N(t) = A \sin(Bt) + C$ .

$$N(t) = 2400 \sin\left(\frac{2\pi}{5}t\right) + 400$$

$$p = 5 \quad \frac{2\pi}{p}$$

$$B = ?$$

**Question 37 is continued on the next page.**

**Score 1:** The student found the correct number of intersections based on their graph.

---

**Question 37**

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**Question 37 continued**

That same lung, when engaged in exercise, has a volume that can be modeled by  $E(t) = 2000 \sin(\pi t) + 3200$ , where  $E(t)$  is volume in mL and  $t$  is time in seconds.

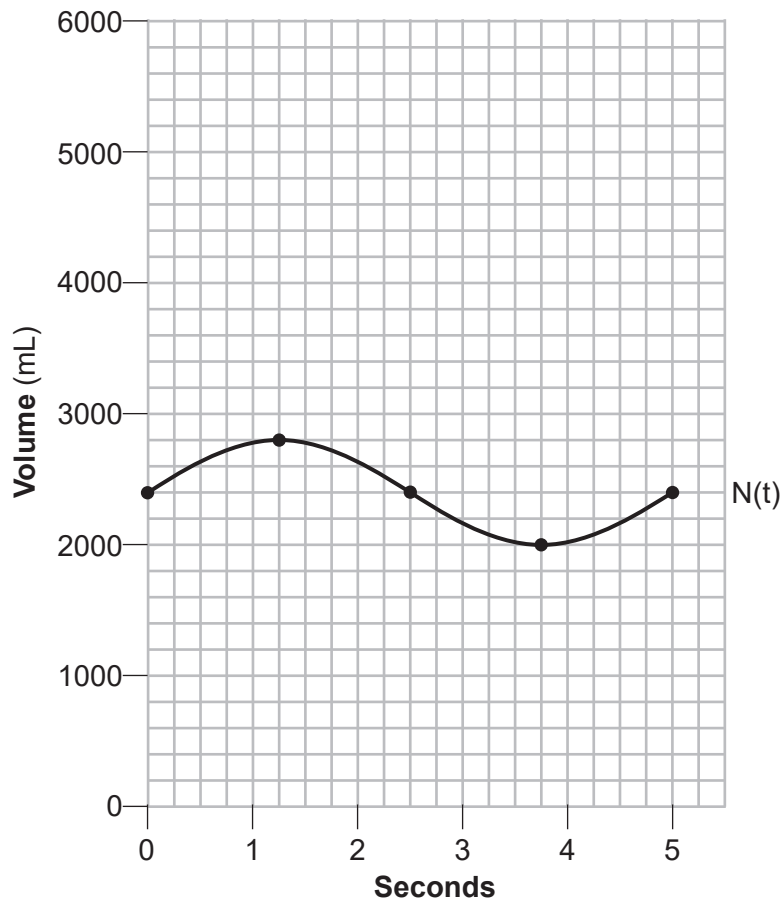
Graph *at least one* cycle of  $E(t)$  on the same grid as  $N(t)$ .

How many times during the 5-second interval will  $N(t) = E(t)$ ?

4

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for  $N(t)$ , in the form  $N(t) = A \sin(Bt) + C$ .

~~$C \sin A = e$~~   
 $2400 \cos(\frac{\pi}{5}x) = N(x)$

Question 37 is continued on the next page.

**Score 0:** The student did not show enough correct work to receive any credit.

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**Question 37**

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**Question 37 continued**

That same lung, when engaged in exercise, has a volume that can be modeled by  $E(t) = 2000 \sin(\pi t) + 3200$ , where  $E(t)$  is volume in mL and  $t$  is time in seconds.

Graph *at least one cycle* of  $E(t)$  on the same grid as  $N(t)$ .

How many times during the 5-second interval will  $N(t) = E(t)$ ?

One time  $N(t) = E(t)$  will  
intersect at the same  
x and y