Stat 250 Gunderson Lecture Notes
7: Learning about a Population Mean

Part 1: Distribution for a Sample Mean

Recall Parameters, Statistics and Sampling Distributions

We go back to the scenario where we have one population of interest but now the response being measured is **quantitative** (not categorical). We want to learn about the value of the **population mean** μ. We take a random sample and use the sample statistic, the **sample mean** $\overline{X}$, to estimate the parameter. When we do this, the sample mean may not be equal to the population mean, in fact, it could change every time we take a new random sample.

So recall that **a statistic is a random variable** and **it will have a probability distribution**. This probability distribution is called the **sampling distribution** of the statistic.

We turn to understanding the **sampling distribution of the sample mean** which will be used to construct a confidence interval estimate for the population mean and to test hypotheses about the value of a population mean.

Sampling Distribution for One Sample Mean

Many responses of interest are measurements – height, weight, distance, reaction time, scores. We want to learn about a population mean and we will do so using the information provided from a sample from the population.

Example: How many hours per week do you work?

A poll was conducted by a Center for Workforce Development. A probability sample of 1000 workers resulted in a mean number of hours worked per week of 43.

Population =

Parameter =

Sample =

Statistic =

Can anyone say how close this observed sample mean $\bar{x}$ of 43 is to the population mean μ?

If we were to take another random sample of the same size, would we get the same value for the sample mean?

So what are the possible values for the sample mean $\bar{x}$ if we took many random samples of the same size from this population? What would the distribution of the possible $\bar{x}$ values look like? What can we say about the **distribution of the sample mean**?

Distribution of the Sample Mean – Main Results

Let μ = mean for the population of interest and σ = standard deviation for the that population.
Let $\bar{x}$ = the sample mean for a random sample of size *n*.

If all possible random samples of the same size *n* are taken and $\bar{x}$ is computed for each, then …

- The average of all of the possible sample mean values is equal to ______________________.

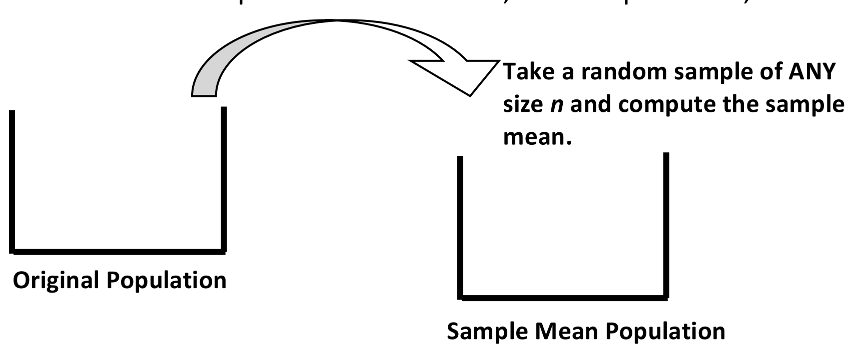
  Thus the sample mean is an ___________________ estimator of the population mean.

- The standard deviation of all of the possible sample mean values is equal to the original population standard deviation divided by $\sqrt{n}$.

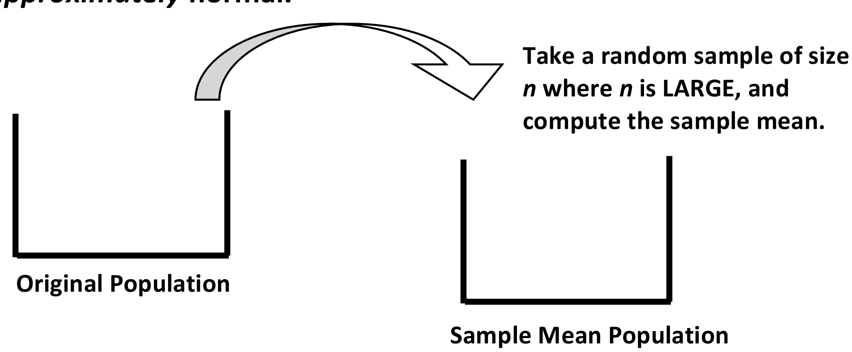
  Standard deviation of the sample mean is given by: s.d.($\bar{x}$ **) =** $\frac{\sigma}{\sqrt{n}}$

What about the shape of the sampling distribution? The first two bullets above provide what the mean and the standard deviation are for the possible sample mean values. The final two bullets tell us that the shape of the distribution will be (approximately) normal.

- If the parent (original) **population has a normal distribution**,
  then the distribution of the possible values of $\bar{x}$, the sample mean, is **normal**.



- If the parent (original) **population is not necessarily normally distributed**
  but the **sample size** n **is large**, then the distribution of the possible values of $\bar{x}$, the sample mean is ***approximately* normal.**



This last result is called the ______________________________________ .

The **C** in CLT is for **CENTRAL**. The CLT is an important or central result in statistics. As it turns out, many normal curve approximations for various statistics are really applications of the CLT. The Stat 250 formula card summarizes distribution of a sample mean as follows:

> **Sample Means**
>
> **Mean**
>
> $E(\overline{X}) = \mu_{\overline{X}} = \mu$
>
> **Standard Deviation**
>
> $s.d.(\overline{X}) = \sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$
>
> **Sampling Distribution of $\overline{X}$**
>
> If X has the $N(\mu, \sigma)$ distribution, then $\overline{X}$ is
>
> $N(\mu_{\overline{X}}, \sigma_{\overline{X}}) \Leftrightarrow N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$.
>
> If X follows *any* distribution with mean μ and standard deviation σ and n is large,
>
> then $\overline{X}$ is *approximately* $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$.
>
> This last result is **Central Limit Theorem**

Try It! SRT Test Scores

A particular test for measuring various aspects of verbal memory is known as the Selective Reminding Task (SRT) test. It is based on hearing, recalling, and learning 12 words presented to the client. Scores for various aspects of verbal memory are combined to give an overall score. Let *X* represent overall score for 20-year-old females. Such scores are normally distributed with a mean of 126 and a standard deviation of 10.

a. What is the probability that a randomly selected 20-year-old female will have a score above 134?

b. A random sample of 9 such females will be selected. What is the probability that all nine will score *below* 134?

c. A random sample of 9 such females will be selected. What is the probability that their sample mean score will be above 134?

Try It! Actual Flight Times

Suppose the random variable *X* represents the actual flight time (in minutes) for Delta Airlines flights from Cincinnati to Tampa follows a *uniform distribution over the range of 110 minutes to 130 minutes.*

a. Sketch the distribution for *X* (include axes labels and some values on the axes).

b. Suppose we were to repeatedly take a random sample of size 100 from this distribution and compute the sample mean for each sample. What would the *histogram of the sample mean values* look like? Provide a smoothed out sketch of the distribution of the sample mean, include all details that you can.

Try It! True or False

Determine whether each of the following statements is True or False. A true statement is always true. Clearly circle your answer.

a. The central limit theorem is important in statistics because for a large random sample, it says the sampling distribution of the sample mean is approximately normal**.**

True ***False***

b. The sampling distribution of a parameter is the distribution of the parameter value if repeated random samples are obtained.

True ***False***

More on the Standard Deviation of $\bar{X}$

The standard deviation of the sample mean is given by: s.d.($\bar{x}$) = $\frac{\sigma}{\sqrt{n}}$

This quantity would give us an idea about how far apart a sample mean $\bar{x}$ and the true population mean μ are expected to be on average.

We can ***interpret*** **the standard deviation of the sample mean** as **approximately** the **average distance** of the possible sample mean value**s** (for repeated samples of the same size *n*) from the **true population mean** μ.

Note: If the sample size increases, the standard deviation decreases, which says the possible sample mean values will be closer to the true population mean (on average).

The **s.d.($\bar{x}$)** is a measure of the ***accuracy of the process*** of using a sample mean to estimate the population mean. This quantity $\frac{\sigma}{\sqrt{n}}$ does not tell us exactly how far away a particular observed $\bar{x}$ value is from μ.

In practice, the population standard deviation σ is rarely known, so the sample standard deviation *s* is used. As with proportions, when making this substitution we call the result the **standard error of the mean s.e.($\bar{x}$) =** $\frac{s}{\sqrt{n}}$. This terminology makes sense, because this is a measure of how much, on ***average***, the sample mean is in error as an estimate of the population mean.

Standard error of the sample mean is given by: s.e.($\bar{x}$) = $\frac{s}{\sqrt{n}}$

This quantity is an estimate of the standard deviation of $\bar{x}$.

So we can ***interpret*** **the standard error of the sample mean** as estimating, **approximately**, the **average distance** of the possible $\bar{x}$ value**s** (for repeated samples of the same size *n*) from the **true population mean** μ.

Moreover, we can use this standard error to create a range of values that we are very confident will contain the true population mean μ, namely, $\bar{x}$ ±(few)s.e.($\bar{x}$). This is the basis for confidence interval for the population mean μ, discussed in Part 2.

Preparing for Statistical Inference: Standardized Statistics

In our SRT Test Scores and Actual Flight Times examples earlier, we have already constructed and used a **standardized *z*-statistic for a sample mean.**

$z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ has (approximately) a standard normal distribution *N*(0,1).

Dilemma = ______________________________________

If we replace the population standard deviation σ with the sample standard deviation s, then $\dfrac{\bar{x}-\mu}{s/\sqrt{n}}$ won't be approximately *N*(0,1); instead it has a ________________________

Student's *t*-Distribution

A little about the family of t-distributions ...

- They are symmetric, unimodal, centered at 0.
- They are flatter with heavier tails compared to the *N*(0,1) distribution.
- As the degrees of freedom (df) increases ... the *t* distribution approaches the *N*(0,1) distribution.
- We can still use the ideas about standard scores for a frame of reference.
- Tables A.2 and A.3 summarize percentiles for various t-distributions

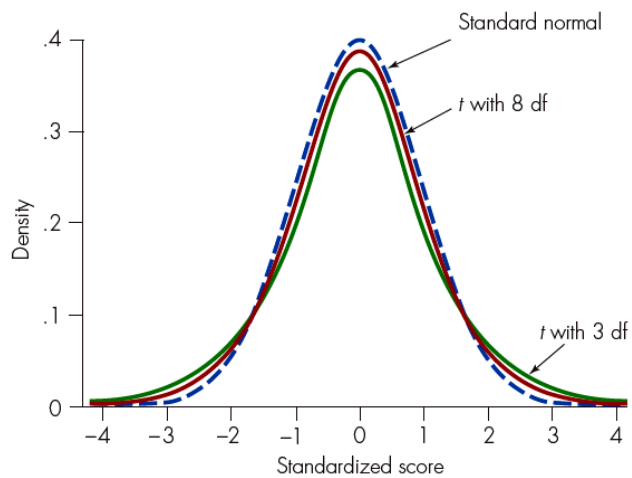


Figure 9.10 ▮ *t*-distributions with df = 3, df = 8, and a standard normal distribution

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We will see more on t-distributions when we do inference about population mean(s).

Every Statistic has a Sampling Distribution

The **sampling distribution of a statistic** is the distribution of possible values of the statistic for repeated samples of the same size from a population.

So far we have discussed the sampling distribution of a sample proportion, the sampling distribution of the difference between two sample proportions, and the sampling distribution of the sample mean. In all cases, under specified conditions the sampling distribution was *approximately* normal.

Every statistic has a sampling distribution, but the appropriate distribution may not always be normal, or even be approximately bell-shaped.

You can construct an approximate sampling distribution for any statistic by actually taking repeated samples of the same size from a population and constructing a histogram for the values of the statistic over the many samples.

Additional Notes

A place to … jot down questions you may have and ask during office hours, take a few extra notes, write out an extra problem or summary completed in lecture, create your own summary about these concepts.

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Part 2: Confidence Interval for a Population Mean

Do not put faith in what statistics say until you have carefully considered
what they do not say. --*William W. Watt* © FAIR USE

Earlier we studied **confidence intervals for estimating a population proportion and the difference between two population proportions**. Recall it is important to understand how to interpret an interval and how to interpret what the confidence level really means.

- The **interval provides a range of reasonable values** for the parameter with an associated high level of confidence. For example we can say, "We are 95% confident that the proportion of Americans who do not get enough sleep at night is somewhere between 0.325 to 0.395, based on a random sample of n = 935 American adults.
- The **95% confidence level describes our confidence in the procedure** we used to make the interval. If we repeated the procedure many times, we would expect about 95% of the intervals to contain the population parameter.

Confidence Interval for a Population Mean μ

Consider a study on the **design of a highway sign**. A question of interest is: What is the **mean** maximum distance at which drivers are able to read the sign? A highway safety researcher will take a random sample of n = 16 drivers and measure the maximum distances (in feet) at which each can read the sign.

Population parameter

μ = __________ **mean** maximum distance to read the sign for __________________

Sample estimate

$\bar{x}$ = __________ **mean** maximum distance to read the sign for __________________

But we know the sample estimate $\bar{x}$ may not equal μ, in fact, the possible $\bar{x}$ values vary from sample to sample. Because the sample mean is computed from a random sample, then it is a random variable, with a probability distribution.

Sampling Distribution of the sample mean

If $\bar{x}$ is the sample mean for a random sample of size n, and either the original population of responses has a normal model or the sample size is large enough,
the distribution of the sample mean is (***approximately***)

So the possible $\bar{x}$ values vary normally around μ with a standard deviation of $\frac{\sigma}{\sqrt{n}}$. The standard deviation of the sample mean, $\frac{\sigma}{\sqrt{n}}$, is roughly the average distance of the possible sample mean values from the population mean μ. Since we don't know the population standard deviation σ, we will use the sample standard deviation s, resulting in the standard error of the sample mean.

Standard Error of the Sample mean

s.e.($\bar{x}$) = where *s* = sample standard deviation

The standard error of $\bar{x}$ estimates, roughly, the average distance of the possible $\bar{x}$ values from μ. The possible $\bar{x}$ values result from considering all possible random samples of the same size *n* from the same population.

So we have our estimate of the population mean, the sample mean $\bar{x}$, and we have its standard error. To make our confidence interval, we need to know the multiplier.

Sample Estimate ± Multiplier x Standard error

The **multiplier for a confidence interval for the population mean is denoted by *t****, which is the value in a **Student's t distribution with df = *n* – 1** such that the area between –*t* and *t* equals the desired confidence level. The value of *t** will be found using Table A.2. First let's give the formal result.

One-sample t Confidence Interval for μ

$$\bar{x} \pm t^{*}\text{s.e.}(\bar{x})$$

where t^* is an appropriate value for a *t*(*n* – 1) distribution.

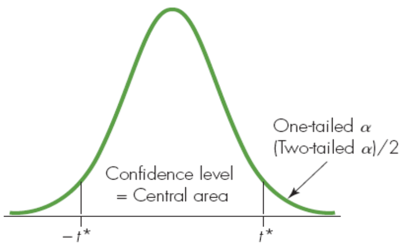
This interval requires we have a random sample from a normal population. If the sample size is large (*n* > 30), the assumption of normality is not so crucial and the result is approximate.

Important items:

- be sure to check the conditions
- know how to interpret the confidence interval
- be able to explain what the confidence level of say 95% really means

Try It! Using Table A.2 to find t^*

Table A.2 t^* Multipliers for Confidence Intervals and Rejection Region Critical Values



	Confidence Level						
df	*.80*	*.90*	*.95*	*.98*	*.99*	*.998*	*.999*
1	3.08	6.31	12.71	31.82	63.66	318.31	636.62
2	1.89	2.92	4.30	6.96	9.92	22.33	31.60
3	1.64	2.35	3.18	4.54	5.84	10.21	12.92
4	1.53	2.13	2.78	3.75	4.60	7.17	8.61
5	1.48	2.02	2.57	3.36	4.03	5.89	6.87
6	1.44	1.94	2.45	3.14	3.71	5.21	5.96
7	1.41	1.89	2.36	3.00	3.50	4.79	5.41
8	1.40	1.86	2.31	2.90	3.36	4.50	5.04
9	1.38	1.83	2.26	2.82	3.25	4.30	4.78
10	1.37	1.81	2.23	2.76	3.17	4.14	4.59
11	1.36	1.80	2.20	2.72	3.11	4.02	4.44
12	1.36	1.78	2.18	2.68	3.05	3.93	4.32
13	1.35	1.77	2.16	2.65	3.01	3.85	4.22
14	1.35	1.76	2.14	2.62	2.98	3.79	4.14
15	1.34	1.75	2.13	2.60	2.95	3.73	4.07
16	1.34	1.75	2.12	2.58	2.92	3.69	4.01
17	1.33	1.74	2.11	2.57	2.90	3.65	3.97
18	1.33	1.73	2.10	2.55	2.88	3.61	3.92
19	1.33	1.73	2.09	2.54	2.86	3.58	3.88
20	1.33	1.72	2.09	2.53	2.85	3.55	3.85
21	1.32	1.72	2.08	2.52	2.83	3.53	3.82
22	1.32	1.72	2.07	2.51	2.82	3.50	3.79
23	1.32	1.71	2.07	2.50	2.81	3.48	3.77
24	1.32	1.71	2.06	2.49	2.80	3.47	3.75
25	1.32	1.71	2.06	2.49	2.79	3.45	3.73
26	1.31	1.71	2.06	2.48	2.78	3.43	3.71
27	1.31	1.70	2.05	2.47	2.77	3.42	3.69
28	1.31	1.70	2.05	2.47	2.76	3.41	3.67
29	1.31	1.70	2.05	2.46	2.76	3.40	3.66
30	1.31	1.70	2.04	2.46	2.75	3.39	3.65
40	1.30	1.68	2.02	2.42	2.70	3.31	3.55
50	1.30	1.68	2.01	2.40	2.68	3.26	3.50
60	1.30	1.67	2.00	2.39	2.66	3.23	3.46
70	1.29	1.67	1.99	2.38	2.65	3.21	3.44
80	1.29	1.66	1.99	2.37	2.64	3.20	3.42
90	1.29	1.66	1.99	2.37	2.63	3.18	3.40
100	1.29	1.66	1.98	2.36	2.63	3.17	3.39
1000	1.282	1.646	1.962	2.330	2.581	3.098	3.300
Infinite	1.281	1.645	1.960	2.326	2.576	3.090	3.291
Two-tailed α	.20	.10	.05	.02	.01	.002	.001
One-tailed α	.10	.05	.025	.01	.005	.001	.0005

(a) Find t^* for a 90% confidence interval based on *n* = 12 observations.

(b) Find t^* for a 95% confidence interval based on *n* = 30 observations.

(c) Find t^* for a 95% confidence interval based on *n* = 54 observations.

(d) What happens to the value of t^* as the sample size (and thus the degrees of freedom) gets larger?

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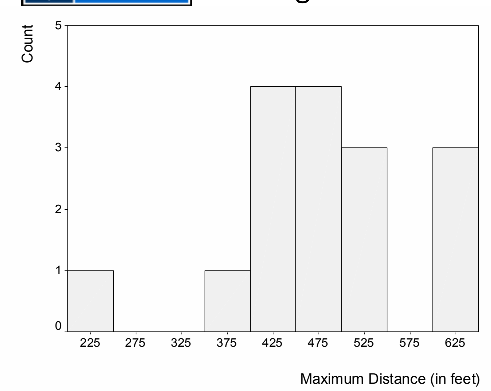
Try It! Confidence Interval for the Mean Maximum Distance

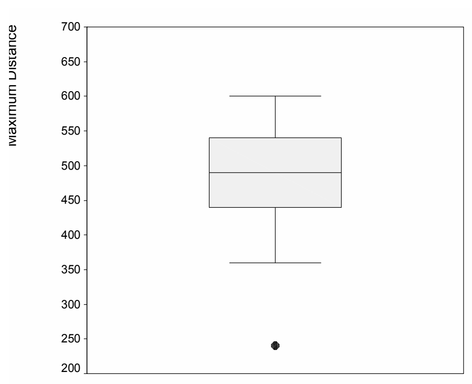
Recall the study on the design of a highway sign. The researcher wanted to learn about the **mean** maximum distance at which drivers are able to read the sign. The researcher took a *random sample* of n = 16 drivers and measured the maximum distances (in feet) at which each can read the sign. The data are provided below.

440	**490**	**600**	**540**	**540**	**600**	**240**	**440**
360	**600**	**490**	**400**	**490**	**540**	**440**	**490**

a. Verify the necessary conditions for computing a confidence interval for the population mean distance. We are told that the sample was a random sample so we just need to check if a normal model for the response 'max distance' for the population is reasonable.

PD-INEL All images





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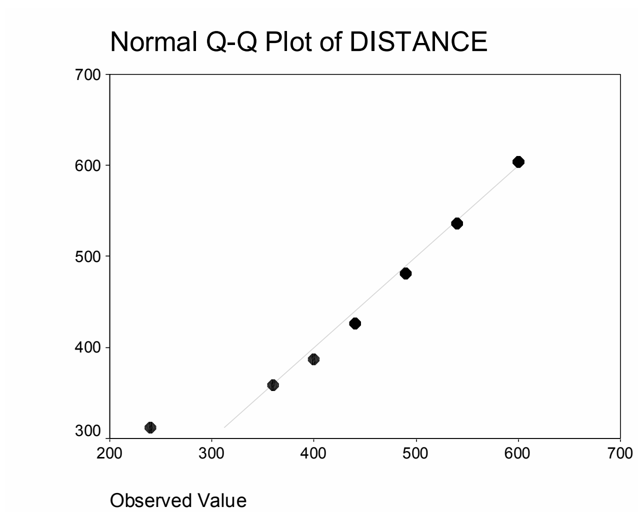
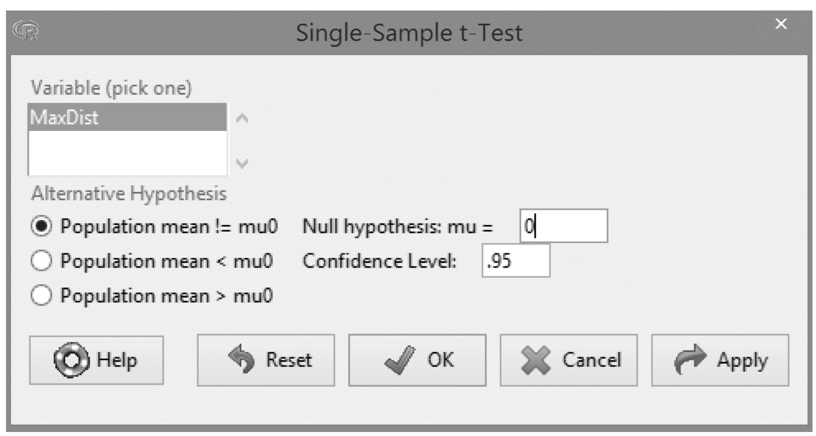


440	**490**	**600**	**540**	**540**	[illegible]	**240**	**440**
360	**600**	**490**	**400**	**490**	**540**	**440**	**490**

b. Compute the sample mean maximum distance and the standard error (without the outlier).

c. Use a 95% confidence interval to estimate the population mean maximum distance at which all drivers can read the sign. Write a paragraph that interprets this interval and the confidence level.

Using R Commander we would use the Single-Sample t-Test to produce the following results. Both the confidence interval and a test of hypotheses will be provided. We will discuss the hypothesis testing for a mean difference in Part 3.




```
      One Sample t-test

data:  MaxDist
t = 20.1005, df = 15, p-value = 2.934e-12
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 430.2184 532.2816
sample estimates:
mean of x
  481.25
```

Additional Notes
A place to … jot down questions you may have and ask during office hours, take a few extra notes, write out an extra problem or summary completed in lecture, create your own summary about these concepts.

Population Mean

Parameter μ

Statistic $\bar{x}$

Standard Error

$$\text{s.e.}(\bar{x}) = \frac{s}{\sqrt{n}}$$

Confidence Interval

$$\bar{x} \pm t^{*}\text{s.e.}(\bar{x}) \qquad \text{df} = n - 1$$

Paired Confidence Interval

$$\bar{d} \pm t^{*}\text{s.e.}(\bar{d}) \qquad \text{df} = n - 1$$

One-Sample *t*-Test

$$t = \frac{\bar{x} - \mu_0}{\text{s.e.}(\bar{x})} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \qquad \text{df} = n - 1$$

Paired *t*-Test

$$t = \frac{\bar{d} - 0}{\text{s.e.}(\bar{d})} = \frac{\bar{d}}{s_d/\sqrt{n}} \qquad \text{df} = n - 1$$

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Part 3: Testing about a Population Mean

Introduction to Hypothesis Tests for Means

We have already been introduced us to the **logic and steps of hypothesis testing for learning about a population proportion and for the difference between two population proportions**. Recall the big idea that we declare "statistical significance" and reject the null hypothesis if the *p*-value is less than or equal to the significance level α. Now we will extend these ideas to testing about means, focusing first on hypothesis testing about a single **population mean**.

A few notes: **Hypotheses and conclusions apply to the larger population(s)** represented by the sample(s). And **if the distribution of a quantitative variable is highly skewed**, we should consider analyzing the median rather than the mean. Methods for testing hypotheses about medians are a special case of **nonparametric methods**, which we will not cover in detail, but do exist as the need arises.

Next let's review the **Basic Steps in Any Hypothesis Test**.

Step 1: Determine the null and alternative hypotheses.
The hypotheses are statements about the population(s), not the sample(s).
The null hypothesis defines a specific value of a population parameter, called the **null value**.

Step 2: Verify necessary data conditions, and if met, summarize the data into an appropriate test statistic.

A relevant statistic is calculated from sample information and summarized into a "test statistic." We measure the difference between the sample statistic and the null value using the standardized statistic:

$$\frac{\textbf{Sample statistic – Null value}}{\textbf{(Null) standard error}}$$

For hypotheses about **proportions** (with large sample sizes), the standardized statistic is called a

_______________ and the _________________________ is used to find the *p*-value.

For hypotheses about **means**, the standardized statistic is called a _______________ and the _________________________ is used to find the *p*-value.

Step 3: Assuming the null hypothesis is true, find the *p*-value.

A *p*-value is computed based on the standardized "test statistic." The *p*-value is calculated by temporarily assuming the null hypothesis to be true and then calculating the probability that the test statistic could be as large in magnitude as it is (or larger) in the direction(s) specified by the alternative hypothesis.

Step 4: Decide if the result is statistically significant based on the *p*-value.

Based on the *p*-value, we either reject or fail to reject the null hypothesis. The most commonly used criterion (level of significance) is that we reject the null hypothesis when the *p*-value is less than or equal to the significance level (generally 0.05). In many research articles, *p*-values are simply reported and readers are left to draw their own conclusions. Remember that a *p*-value measures the strength of the evidence against the null hypothesis and the smaller the *p*-value, the stronger the evidence against the null (and for the alternative).

The *Beauty* of *p*-values: Suppose the significance level α is set at 5% for testing H_0: status quo versus H_a: the "new theory".

If *p*-value is…	Statistical Decision	Feasible Conclusion about the "New Theory"
0.462	**Fail to Reject H_0**	
0.063	**Fail to Reject H_0**	
0.041	**Reject H_0**	
0.003	**Reject H_0**	

Step 5: Report the conclusion in the context of the situation.

The decision is to reject or fail to reject the null hypothesis, but the conclusion should go back to the original question of interest being asked. It should be stated in terms of the particular scenario or situation.

Testing Hypotheses about One Population Mean μ

We have **one population** and a **response that is quantitative**. We wish to test about the value of the **mean response for the population** μ.. The data are assumed to be a **random sample**. The response is assumed to be **normally distributed** for the population (but if the sample size is large, this condition is less crucial).

Step 1: Determine the null and alternative hypotheses.

1. **H_0:** **versus H_a:**

2. **H_0:** **versus H_a:**

3. **H_0:** **versus H_a:**

Step 2: Verify necessary data conditions, and if met, summarize the data into an appropriate test statistic.

How would you check the conditions as stated in the scenario above?

Test statistic = $\dfrac{\text{Sample statistic} - \text{Null value}}{\text{Standard error}}$

If H_0 is true, this test statistic has a _______________ distribution.

Step 3: Assuming the null hypothesis is true, find the *p*-value.

Steps for finding a *p*-value …

- **Draw the distribution for the test statistic under H_0**
  For t tests it will be a t-distribution with a certain df.
- **Locate the observed test statistic value on the axis.**
- **Shade in the area that corresponds to the *p*-value.**
  Look at the alternative hypothesis for the direction of extreme.
- **Use the appropriate table to find (bounds for) the *p*-value.**
  For t tests we use Table A.3.

Step 4: Decide whether or not the result is statistically significant based on the *p*-value.

The level of significance α is selected in advance. **We reject the null hypothesis if the *p*-value is less than or equal to α..** In this case, we say the results are statistically significant at the level α.

Step 5: Report the conclusion in the context of the situation.

Once the decision is made, a conclusion in the context of the problem can be stated.

From the Stat 250 formula card:

Population Mean	
Parameter	μ
Statistic	$\bar{x}$
Standard Error $\text{s.e.}(\bar{x}) = \dfrac{s}{\sqrt{n}}$	

One-Sample *t*-Test

$$t = \frac{\bar{x} - \mu_0}{\text{s.e.}(\bar{x})} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \qquad \text{df} = n - 1$$

TABLE A.3 ■ One-Sided *p*-Values for Significance Tests Based on a *t*-Statistic

- A *p*-value in the table is the area to the right of *t*.
- Double the value if the alternative hypothesis is two-sided (not equal).

	Absolute Value of *t*-Statistic							
df	*1.28*	*1.50*	*1.65*	*1.80*	*2.00*	*2.33*	*2.58*	*3.00*
1	0.211	0.187	0.173	0.161	0.148	0.129	0.118	0.102
2	0.164	0.136	0.120	0.107	0.092	0.073	0.062	0.048
3	0.145	0.115	0.099	0.085	0.070	0.051	0.041	0.029
4	0.135	0.104	0.087	0.073	0.058	0.040	0.031	0.020
5	0.128	0.097	0.080	0.066	0.051	0.034	0.025	0.015
6	0.124	0.092	0.075	0.061	0.046	0.029	0.021	0.012
7	0.121	0.089	0.071	0.057	0.043	0.026	0.018	0.010
8	0.118	0.086	0.069	0.055	0.040	0.024	0.016	0.009
9	0.116	0.084	0.067	0.053	0.038	0.022	0.015	0.007
10	0.115	0.082	0.065	0.051	0.037	0.021	0.014	0.007
11	0.113	0.081	0.064	0.050	0.035	0.020	0.013	0.006
12	0.112	0.080	0.062	0.049	0.034	0.019	0.012	0.006
13	0.111	0.079	0.061	0.048	0.033	0.018	0.011	0.005
14	0.111	0.078	0.061	0.047	0.033	0.018	0.011	0.005
15	0.110	0.077	0.060	0.046	0.032	0.017	0.010	0.004
16	0.109	0.077	0.059	0.045	0.031	0.017	0.010	0.004
17	0.109	0.076	0.059	0.045	0.031	0.016	0.010	0.004
18	0.108	0.075	0.058	0.044	0.030	0.016	0.009	0.004
19	0.108	0.075	0.058	0.044	0.030	0.015	0.009	0.004
20	0.108	0.075	0.057	0.043	0.030	0.015	0.009	0.004
21	0.107	0.074	0.057	0.043	0.029	0.015	0.009	0.003
22	0.107	0.074	0.057	0.043	0.029	0.015	0.009	0.003
23	0.107	0.074	0.056	0.042	0.029	0.014	0.008	0.003
24	0.106	0.073	0.056	0.042	0.028	0.014	0.008	0.003
25	0.106	0.073	0.056	0.042	0.028	0.014	0.008	0.003
26	0.106	0.073	0.055	0.042	0.028	0.014	0.008	0.003
27	0.106	0.073	0.055	0.042	0.028	0.014	0.008	0.003
28	0.106	0.072	0.055	0.041	0.028	0.014	0.008	0.003
29	0.105	0.072	0.055	0.041	0.027	0.013	0.008	0.003
30	0.105	0.072	0.055	0.041	0.027	0.013	0.008	0.003
40	0.104	0.071	0.053	0.040	0.026	0.012	0.007	0.002
50	0.103	0.070	0.053	0.039	0.025	0.012	0.006	0.002
60	0.103	0.069	0.052	0.038	0.025	0.012	0.006	0.002
70	0.102	0.069	0.052	0.038	0.025	0.011	0.006	0.002
80	0.102	0.069	0.051	0.038	0.024	0.011	0.006	0.002
90	0.102	0.069	0.051	0.038	0.024	0.011	0.006	0.002
100	0.102	0.068	0.051	0.037	0.024	0.011	0.006	0.002
1000	0.100	0.067	0.050	0.036	0.023	0.010	0.005	0.001
Infinite	0.1003	0.0668	0.0495	0.0359	0.0228	0.0099	0.0049	0.0013

Note that the *t*-distribution with infinite df is the standard normal distribution.

Try It! Using Table A.3 to find a *p*-value for a one-sided test

We are testing H_0: $\mu = 0$ versus H_a: $\mu > 0$ with $n = 15$ observations and the observed test statistic is $t = 1.97$

- **Draw the distribution for the test statistic under H_0**

- **Locate the observed test statistic value on the axis.**
- **Shade in the area that corresponds to the *p*-value.**
  Look at the alternative hypothesis for the direction of extreme.
- **Use the appropriate table to find (bounds for) the *p*-value.**
  For t tests we will use Table A.3.

Is the value of $t = 1.97$ significant at the 5% level? __________

At the 1% level? _________

Try It! Using Table A.3 to find a *p*-value for a two-sided test

We are testing H_0: $\mu = 64$ versus H_a: $\mu \neq 64$ with $n = 30$ observations and the observed test statistic is $t = 1.12$. How would you report the *p*-value for this test?

Try It! Classical Music

A researcher wants to test if HS students complete a maze more quickly while listening to classical music. For the general HS population, the time to complete the maze is assumed to follow a normal distribution with a mean of 40 seconds. Use a 5% significance level.

Define the parameter of interest: Let μ represent…

State the hypotheses: **H_0:**

H_a:

A random sample of 100 HS students are timed while listening to classical music.
The mean time was 39.1 seconds and the standard deviation was 4 seconds. Conduct the test.

Are the results statistically significant at the 5% level? ________________________
State the conclusion at the 5% level in terms of the problem.

Comment about the assumptions required for this test to be valid:

Try It! Calcium Intake

A bone health study looked at the daily intake of calcium (mg) for 38 women. They are concerned that the mean calcium intake for the population of such women is not meeting the RDA level of 1200 mg, that is, the population mean is less than the 1200 mg level. They wish to test this theory using a 5% significance level.

a. State the hypotheses about the mean calcium intake for the population of such women.

H_0: _________________________ versus H_a: ______________________

Summary Statistics			
Mean	**Std. Dev (s)**	**Sample Size (n)**	**Std. Error**
926.03	427.23	38	69.31

Below are the *t*-test results generated using **R Commander** and selecting **Statistics > Means > Single-Sample T Test**. A test value of 1200 was entered and the correct direction for the alternative hypothesis was selected. Notice that a 95% one-sided confidence bound is provided since our test alternative was one-sided to the left. If you wanted to also report a regular 95% confidence interval, you would run a two-sided hypothesis test in R.

One Sample T Results				
t	***df***	***p-value***	***95% CI Lower***	***95% CI Upper***
-3.953	37	0.000165	***	1043.16

b. Interpret the Std. Error of the mean (SEM):

c. Give the observed test statistic value: _____ = _______________
Interpret the this value in terms of a difference from the hypothesized mean of 1200.

d. Sketch a picture of the *p*-value in terms of an area under a distribution.

e. Give the *p*-value and the conclusion using a 5% significance level.

The Relationship between Significance Tests and Confidence Intervals

Earlier we discussed the using of confidence intervals to guide decisions. A confidence interval provides a **range of plausible (reasonable) values** for the parameter. The null hypothesis gives a **null value** for the parameter. So:

- **If this null value is one of the "reasonable" values** found in the confidence interval, the **null hypothesis would not be rejected**.
- **If this null value was not found in the confidence interval** of acceptable values for the parameter, then **the null hypothesis would be rejected**.

Notes:

(1) The alternative hypothesis should be **two-sided**. However, sometimes you can reason through the decision for a one-sided test.

(2) The **significance level of the test should coincide with the confidence level** (e.g. α = 0.05 with a 95% confidence level). However, sometimes you can still determine the decision if these do not exactly correspond (see part (c) of the next Try It!).

(3) This relationship holds exactly for tests about a population mean or difference between two population means. In most cases, the correspondence will hold for tests about a population proportion or difference between two population proportions.

Try It! Time Spent Watching TV

A study looked at the amount of time that teenagers are spending watching TV. Based on a representative sample, the 95% confidence interval for mean amount of time (in hours) spent watching TV on a weekend day was given as: 2.6 hours ± 2.1 hours. So the interval goes from 0.5 hours to 4.7 hours.

a. Test H_0: μ = 5 hours versus H_a: $\mu \neq$ 5 hours at $\alpha = 0.05$.
Reject H_0 Fail to reject H_0 Can't tell
Why?

b. Test H_0: μ = 4 hours versus H_a: $\mu \neq$ 4 hours at $\alpha = 0.05$.
Reject H_0 Fail to reject H_0 Can't tell
Why?

c. Test H_0: μ = 4 hours versus H_a: $\mu \neq$ 4 hours at $\alpha = 0.01$
Reject H_0 Fail to reject H_0 Can't tell
Why?

d. Test H_0: μ = 4 hours versus H_a: $\mu \neq$ 4 hours at $\alpha = 0.10$

Reject H_0 Fail to reject H_0 Can't tell
Why?

Try It! MBA grads Salaries

"It's a good year for MBA grads" was the title of an article. One of the parameters of interest was the population mean expected salary, μ (in dollars). A random sample of 1000 students who finished their MBA this year (from 129 business schools) resulted in a 95% confidence interval for μ of (83700, 84800).

a. What is the value of the sample mean? Include your units.

b. For each statement determine if it is true or false. ***Clearly circle your answer.***

If repeated samples of 1000 such students were obtained, we would expect 95% of the resulting intervals to contain the population mean.

True **False**

There is a 95% probability that the population mean lies between $83,700 and $84,800.

True **False**

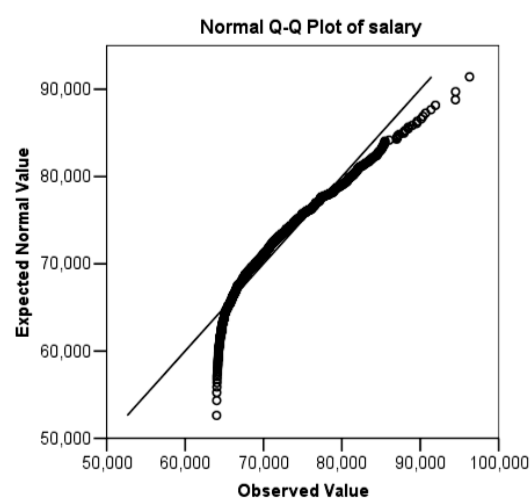
c. The expected average earnings for such graduates in past year was $76,100. Suppose we wish to test the following hypotheses at the *10% significance level*:

H_0: μ = 76100 versus H_a: $\mu \neq$ 76100.

Our decision would be: **Fail to reject** H_0 **Reject** H_0 **can't tell**

Because …

d. Several plots of the expected salary data were constructed to help verify some of the data conditions. A qq-plot is provided for checking the assumption that the response is normally distributed. This plot shows some departure from a straight line with a positive slope. Is this cause for concern that inference based on our confidence interval and hypothesis test would not be valid? Explain.



Additional Notes

A place to … jot down questions you may have and ask during office hours, take a few extra notes, write out an extra problem or summary completed in lecture, create your own summary about these concepts.

Population Mean

Parameter	μ
Statistic	$\bar{x}$

Standard Error

$$\text{s.e.}(\bar{x}) = \frac{s}{\sqrt{n}}$$

Confidence Interval

$$\bar{x} \pm t^{*}\text{s.e.}(\bar{x}) \qquad \text{df} = n - 1$$

Paired Confidence Interval

$$\bar{d} \pm t^{*}\text{s.e.}(\bar{d}) \qquad \text{df} = n - 1$$

One-Sample *t*-Test

$$t = \frac{\bar{x} - \mu_0}{\text{s.e.}(\bar{x})} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \qquad \text{df} = n - 1$$

Paired *t*-Test

$$t = \frac{\bar{d} - 0}{\text{s.e.}(\bar{d})} = \frac{\bar{d}}{s_d/\sqrt{n}} \qquad \text{df} = n - 1$$