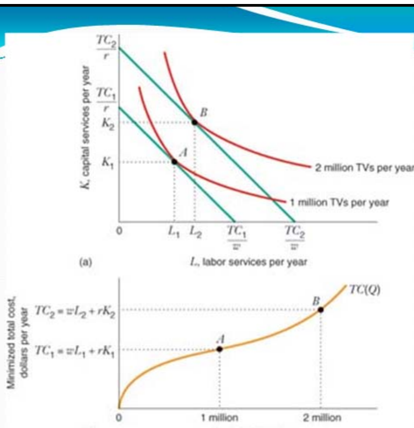
Costs Curves

Chapter 8, Lecture slides

Long Run Total Cost

- The long run total cost curve shows the total cost of a firm's optimal choice combinations for labor and capital as the firm's total output increases.
- Note that the total cost curve will always be zero when Q=0 because in the long run a firm is free to vary all of its inputs.



$\Delta Q \rightarrow \Delta TC$

$Q = 0 \rightarrow TC = 0$

Example

- Suppose that the production function is…

$$Q = 50\sqrt{LK}$$

- We know from chapter 7 that the cost-minimizing amount of labor and capital that the firm will demand are…

$$L = \frac{Q}{50}\left(\frac{r}{w}\right)^{\frac{1}{2}} \;\&\; K = \frac{Q}{50}\left(\frac{w}{r}\right)^{\frac{1}{2}}$$

So…

Total costs becomes

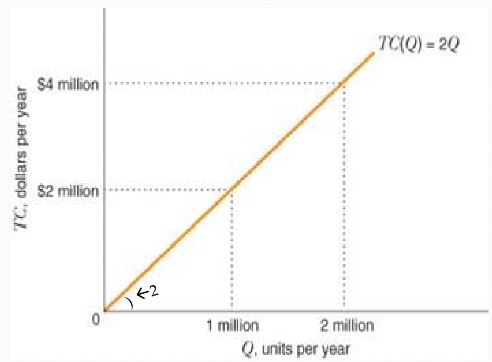
$$TC = wL + rK \Rightarrow w\frac{Q}{50}\left(\frac{r}{w}\right)^{\frac{1}{2}} + r\frac{Q}{50}\left(\frac{w}{r}\right)^{\frac{1}{2}} \Rightarrow \frac{Q}{50}(rw)^{0.5} + \frac{Q}{50}(rw)^{0.5} \Rightarrow 2\frac{Q}{50}(rw)^{0.5}$$

$$TC = \frac{Q}{25}(rw)^{0.5}$$

If we are told that w=25 and r=100, then these total costs are

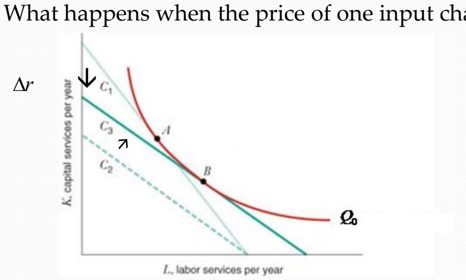
$$TC(Q) = \frac{Q}{25}(25*100)^{0.5} = \left(\frac{Q}{25}\right)50 = 2Q$$

which is a straight line (as depicted in the next slide)



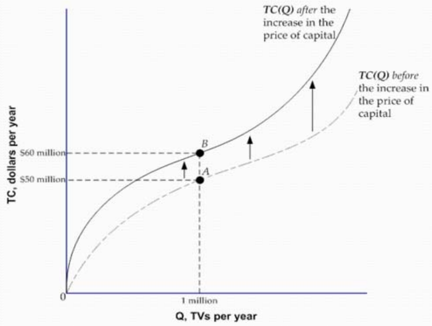
Input Price Changes

- What happens when the price of one input changes?



- Analysis of graph…
  - Δr forces the isocost line C_1 to pivot to left (flatter) since K is more expensive, but TC remain constant because the isocost doesn't change.
  - However, the Q_0 output level must remain unchanged, then isocost C_2 shifts outward up to C_3, so this firm incurs a higher total cost to maintain the Q_0 level of output
- Hence, the change in r forces a change in TC (ΔTC)

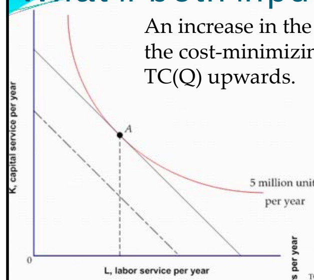
Thus, an increase in the price of one input (e.g. capital) shifts $TC(Q)$ upwards

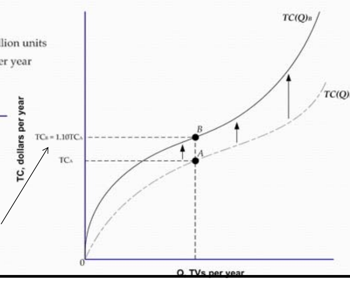


What if both input prices change?

An increase in the price of both inputs does not affect the cost-minimizing input combination, but shifts TC(Q) upwards.

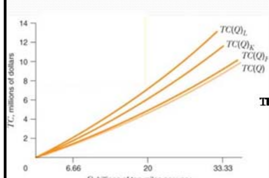
Example: $\frac{w}{r} \to \Delta 10\% \to \frac{\cancel{1.1}w}{\cancel{1.1}r}$



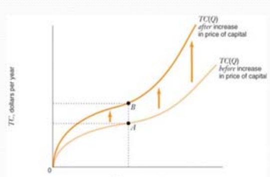


TC_B is now 10% larger than TC_A

<u>Application 8.1</u> Increasing input price in U.S. Bus Systems



Then Δr implies ⇒



When $\Delta r = \Delta w$ then the K/L ratio doesn't change because the tradeoff between the two inputs doesn't change, but total costs will increase. In particular, $\Delta TC(Q) = \Delta r = \Delta w$

An increase in w produces a shift from TC(Q) to $TC(Q)_L$

An increase in r or the price of fuel produces a smaller shift from TC(Q) to $TC(Q)_K$ or $TC(Q)_F$

} Labor costs are 50% of TC in a typical bus system

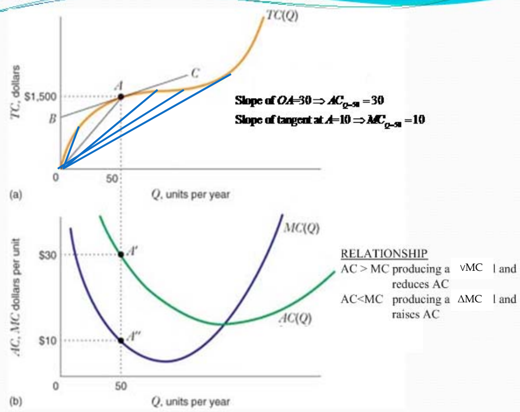
Long Run Average and Marginal Cost

- The **LRAC** is equivalent to the slope of any ray from the origin to a point on the TC curve (see next slide).

$$LRAC = \frac{TC(Q)}{Q}$$

- The **LRMC** is equal to the slope of a line tangent to the TC curve (see next slide).
- The LRMC tells us how the total cost changes as a firm increases output by one unit.

$$LRMC = \frac{dTC(Q)}{dQ}$$



- That is, when additional units are cheaper and cheaper to produce (in the declining portion of the marginal cost curve), the firm's average costs will also be decreasing.

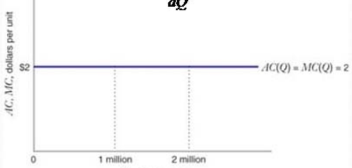
Example

- Given the production function…

  $Q = 50\sqrt{LK}$ and w=25 $r = 100$ we have $TC(Q) = 2Q$
- Let's find the corresponding the AC and MC curves:

$$AC = \frac{TC}{Q} = \frac{2Q}{Q} = 2$$

$$MC = \frac{dTC}{dQ} = 2$$



When the TC(Q) is a straight line then LRAC=LRMC

Query #1

Suppose a firm's short run total cost curve can be expressed as $STC = 50Q + 10$. This firm's short-run marginal cost can be expressed as

a) $50 + 10/Q$

b) $50Q$

c) 50

d) 10

Query #1 - Answer

- Answer C
- The Short-Run Marginal Cost is equal to the slope of the Short-Run Total Cost at a particular point, so simply take the derivative of the Short-Run Total Cost function, 50Q+10, with respect to Q,

$$\frac{\partial(50Q+10)}{\partial Q} = 50$$

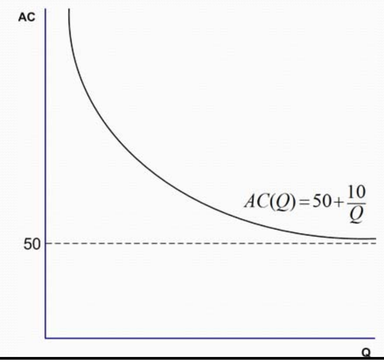
- Page 305

Query #1 - Answer

As a curiosity, note that the AC curve will be given by:

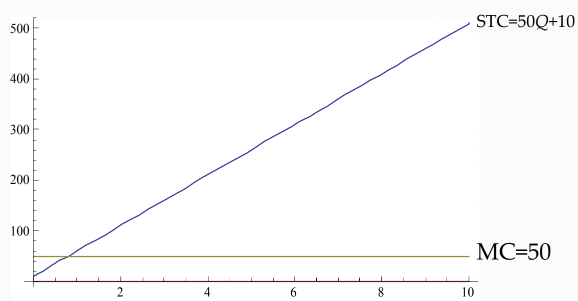
$$AC(Q) = \frac{TC(Q)}{Q}$$

$$= \frac{50Q+10}{Q} = 50 + \frac{10}{Q}$$



Query #1 - Answer

Let's now depict the total cost curve, and the marginal cost curve



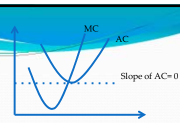
Query #2

Suppose a firm produces 50,000 units of output, and determines that its marginal cost is \$0.72 and its average total cost is \$0.72.

At this quantity of output, what is the slope of this firm's long run average total cost curve?

a) Positive.
b) Negative.
c) Zero.

Query #2 - Answer



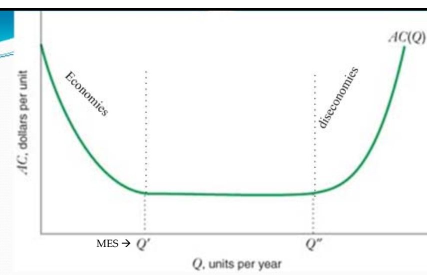
- Answer C
- At the point where the Long-Term Average Cost Curve and the Long-Term Marginal Cost Curve intersect, the Long-Term Average Cost Curve is at its minimum.
- Therefore, the slope of the Long-Term Average Cost Curve is *zero*, or horizontal.
- Pages 292-294

Economies of Scale

- **Economies of Scale** – Property by which the firm's AC *decreases* as output increases.
  - They arise from specialization, but also from <u>indivisible inputs</u>: an input that is available only in a certain minimum size. Its quantity cannot be scaled down as the firm's output goes to zero. In this case, the cost of producing a very small amount of output is very similar to the cost of producing a very large output level.
    - **<u>Example</u>:** High-speed packaging machine for a breakfast cereal, where the smallest of them produces 14 million pounds of breakfast cereal per year!

Economies of Scale

- **Diseconomies of Scale** – Property by which the firm's AC *increases* when output decreases.
  - They arise from <u>managerial diseconomies</u>: a given %ΔQ forces the firm to largely increase its spending on managers by more than this percentage.



MES as a % of total industry output → $\frac{MES}{Q} = \begin{cases} \text{High for breakfast cereal and cane sugar} \\ \text{Low for bread} \end{cases}$

- ***Minimum Efficient Scale*** - minimum Q for which the firm attains its minimum point (on the above graph), the leftward most point on the straight line of the AC.

Relationship between Economies of Scale and Returns to Scale

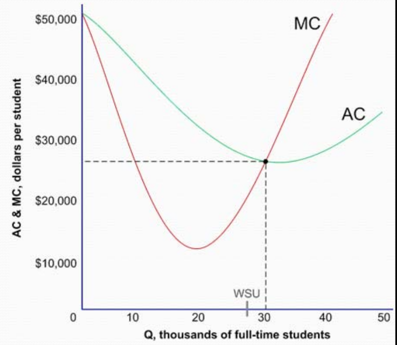
	Production Function		
	$Q=L^2$	$Q=\sqrt{L}$	$Q=L$
Labor demand (found by doing cost minimization)	$L=\sqrt{Q}$	$L=Q^2$	$L=Q$
Long-run total cost	$TC=w\sqrt{Q}$	$TC=wQ^2$	$TC=wQ$
Long-run average cost	$AC=w\sqrt{Q}$	$AC=wQ$	$AC=w$
How does long-run average cost vary with Q?	Decreasing	Increasing	Constant
Economies/diseconomies of scale?	Economies of Scale	Diseconomies of Scale	Neither
Returns to scale (found by increasing all inputs by λ).	Increasing	Decreasing	Constant

Average and Marginal Cost per Undergraduate in 66 top universities

Most universities lie below the minimum of the AC curve.

This is because large fixed costs of running the university (libraries, buildings, stadiums, etc.) are spread over a large student population, whereas variable costs don't increase substantially due to more students.

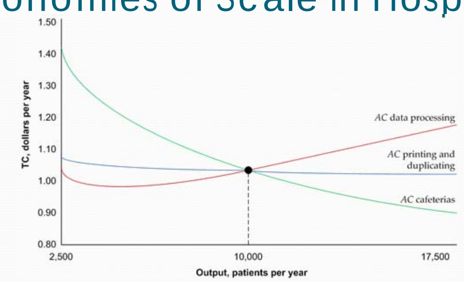
Implication: Increase the student population!



Hospital Mergers and Economies of Scale

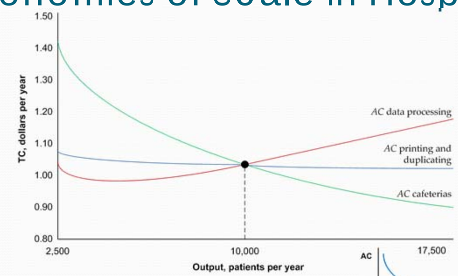
- In the 1990's several hospitals merged:
  - *Good because*: Cost savings through economies of scale in "Back-office" operations, such as laundry, housekeeping, printing, data processing, etc.
  - *Or Bad because*: Cost savings are negligible, and thus the merger would only reduce competition
  - To answer the question we need to measure the size of the economies of scale in the above services →

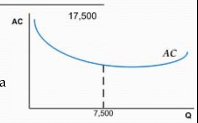
Economies of Scale in Hospitals



- Hospital cafeterias (green curve) exhibit significant economies of scale.
- Printing and duplicating administrative activities (blue curve) are essentially flat, i.e., no economies of scale.
- Data processing (red curve) only exhibits eco. of scale at low levels of output.

Economies of Scale in Hospitals





- Combining the 14 main activities of hospitals, eco. of scale are only significant for Q<7,500 patients (around 200 beds), which is a medium-size hospital by today's standards.
- Therefore, a merger of hospitals would not usually result in reductions for average patient costs.

Query #3

For a firm, let total cost be $TC(Q) = 160+10Q^2$. Its marginal cost is then $MC(Q) = 20Q$.

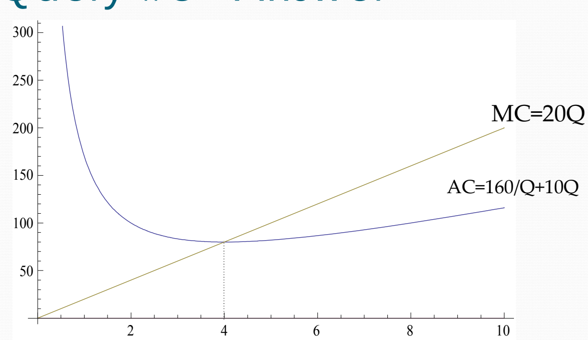
What is the minimum efficient scale (MES) for this firm?

a) 0
b) 2
c) 4
d) Indeterminate

Query #3 - Answer

- Answer C
- The minimum efficient scale is the smallest quantity at which the long-run average cost curve attains its minimum point.
- First, divide the Total Cost Function by Q to find the Average Cost
  - $AC=[160+10Q^2]/(Q)$
- This gives you $AC = 160/Q + 10Q$
- Then, set this equal to marginal cost, because we know that where AC=MC, the AC is at its minimum.
  - $160/Q + 10Q = 20Q$
- After some rearranging,
  - $160/Q = 10Q$
  - $16 = Q^2$
  - $Q=4$
- *Alternative approach:* You can also solve this problem by equating the first order derivative of the average cost function, $AC=[160+10Q^2]/(Q)$, to 0, and solving for Q.
- Pages 297-298

Query #3 - Answer



Minimum Efficient Scale (Q=4 units)

Output Elasticity to Total Cost

- The percentage change in total cost per 1 percent change in output

$\Delta 1\%$ in output $\Rightarrow$ $\%\Delta$ in TC measured by $\varepsilon_{TC,Q}$

- (the same concept behind all elasticities).

$$\varepsilon_{TC,Q} = \frac{\frac{\Delta TC}{TC}}{\frac{\Delta Q}{Q}} = \frac{\Delta TC}{\Delta Q} \cdot \frac{Q}{TC}$$

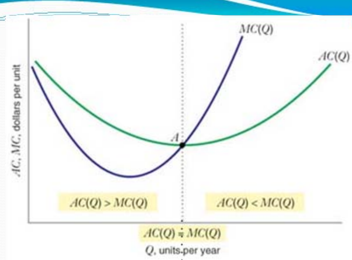
Output Elasticity to Total Cost

$$\varepsilon_{TC,Q} = \frac{\frac{\Delta TC}{TC}}{\frac{\Delta Q}{Q}} = \frac{\Delta TC}{\Delta Q} \cdot \frac{Q}{TC}$$

- Since $MC = \frac{\Delta TC}{\Delta Q}$, and $AC = \frac{TC}{Q}$, then

$$\varepsilon_{TC,Q} = MC * \frac{1}{AC} = \frac{MC}{AC}$$

- This is a very convenient formula, as it is a function of *MC* and *AC* alone.



$\varepsilon_{TC,Q} = \frac{MC}{AC}$

$MC < AC$

$\varepsilon_{TC,Q} < 1$
Economies of scale
Examples:
Iron, Steel
Electricity
Gas

$\varepsilon_{TC,Q} > 1$
Diseconomies of scale
Examples:
Textile,
Cement

$\varepsilon_{TC,Q} = \frac{MC}{AC}$

$MC > AC$

Estimates of $\varepsilon_{TC,Q}$

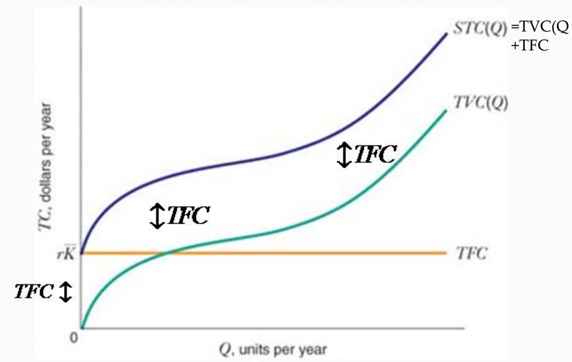
Electric Power Generation		**Four Computer Industries**	
All utilities	.993	Computers	.759
Nuclear Utilities	.995	Computer storage services	.652
Non-nuclear utilities (fossil fuels)	.992	Computer terminals	.636
		Computer Peripheral Equipment	.664

Δ 1% in ouput $\Rightarrow$ Δ less than 1% in total costs

Short Run Total Cost

- One or more inputs are fixed at given level
- Unlike in LRTC, where all inputs can vary so the firm is left with complete freedom in choosing between inputs.
  - In the short run, we can split the cost between the cost that varies (Total Variable Cost) and the cost that is fixed, Total Fixed Cost (hence short run).

Dividing Total Costs between Variable and Fixed



Example

- Suppose that the production function is, but K is fixed at $\bar{K}$ where w=25 and r=100…

$$Q = 50\sqrt{LK}$$

- Solving for L to get the cost-minimizing amount of labor…

$$Q = 50\sqrt{L\bar{K}} \Rightarrow (Q)^2 = \left(50\sqrt{L\bar{K}}\right)^2 \Rightarrow Q^2 = 2500L\bar{K} \Rightarrow \boxed{L = \frac{Q^2}{2500\bar{K}}}$$

- Plugging it into the TC, we find the short-run total costs

L from previous slide

$$STC(Q) = wL + r\bar{K} \Rightarrow 25\frac{Q^2}{2500\bar{K}} + 100\bar{K} \Rightarrow \underbrace{\frac{Q^2}{100\bar{K}}}_{TVC} + \underbrace{100\bar{K}}_{TFC}$$

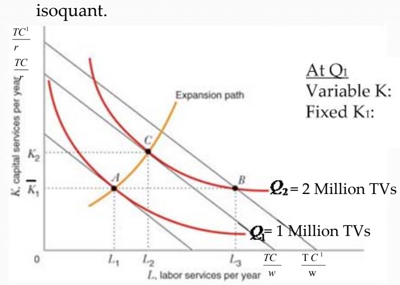
where TVC $= \dfrac{Q^2}{100k}$ is Increasing in Q, and Decreasing in k

and TFC = $100\bar{k}$ is increasing in $\bar{k}$ but remains <u>constant</u> in Q

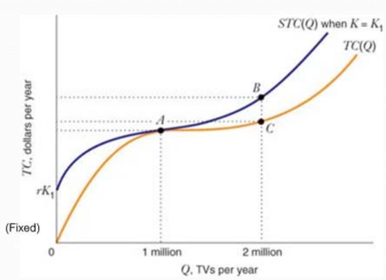
Short Run Total Cost vs Long Run Total Cost

- When the firm is free to vary the quantity of capital in the long run, it can attain lower total cost than it can when its capital is fixed.

- Point B is the short run optimal basket and C is the long run optimal basket.
- Notice that point B costs more than point C and is on the same isoquant.



At Q_1
Variable K: Firm will choose K_1
Fixed K_1: Firm will choose L_1 and precisely K_1



At Q_2
Variable K: Firm will choose K_2, L_2
Fix $\bar{K}$: Firm will choose L_3 with $\bar{K}$ but at a higher isocost than of point C

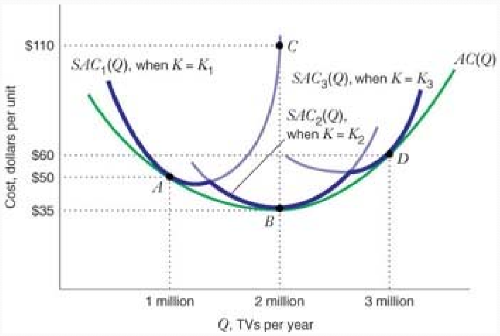
Short Run Average Costs

- Same concept as LRAC, but the cost function includes the fixed inputs…

e.g. Labor e.g. Taxes

$$SAC = \frac{STC(Q)}{Q} = \frac{VC + FC}{Q} = \frac{VC}{Q} + \frac{FC}{Q} = AVC + AFC$$

$$= \frac{(w \cdot L) + (r \cdot \bar{K}_1)}{Q}$$



Long Run Average Cost

- Consider the production function…

$$Q = 50\sqrt{LK}$$

- What is the SAC for a fixed $\overline{K}$ when w=25 and r=100?
  - First, from previous exercises we know that the short-run total cost function of this production function is

$$STC(Q) = \frac{Q^2}{100\overline{K}} + 100\overline{K}$$

Thus, SAC(Q) is equal to

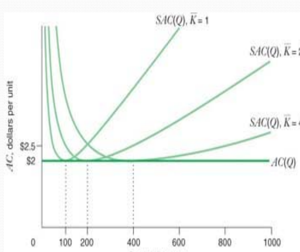
$$\frac{STC}{Q} = SAC(Q) = \frac{Q}{100\overline{K}} + \frac{100\overline{K}}{Q}$$

- Sketch SAC(Q) for the varying levels of fixed capital $\overline{K} = \{1,2,4\}$

$$\overline{K} = 1 \Rightarrow SAC(Q) = \frac{Q}{100} + \frac{100}{Q}$$

$$\overline{K} = 2 \Rightarrow SAC(Q) = \frac{Q}{200} + \frac{200}{Q}$$

$$\overline{K} = 4 \Rightarrow SAC(Q) = \frac{Q}{400} + \frac{400}{Q}$$



- Comparing them with the long-run average cost, *LRAC(Q)=2*, i.e., *TC(Q)=2Q*, we find that *LRAC(Q)* is the lower envelope of the *SAC(Q)*.

Railroad Costs – Application 8.5

A 10 % increase in…	Changes TVC by…
Volume of output	+3.98
Track Mileage	-.271%
Speed of service	-.66%
Price of fuel	+1.90%
Price of Labor	+5.25%
Price of Equipment	+2.85%

Economies of Scope

- Here, the firm produces two products, and the total cost of one single firm that produces these two goods is less than the total cost of producing those quantities in two single product firms.

$$TC(Q_1, Q_2) < TC(Q_1, 0) + TC(0, Q_2)$$

And since $TC(0,0) = 0$, we can rewrite the above inequality as follows: □ 0

$$\Leftrightarrow \underbrace{TC(Q_1,Q_2) - TC(Q_1,0)}_{\text{Additional cost of producing } Q_2 \text{ when firm was only producing } Q_1} < \underbrace{TC(0,Q_2) - TC(0,0)}_{\text{Additional cost of producing } Q_2 \text{ when firm was not producing anything}}$$

Example: Offering one more channel in a satellite TV network is cheaper for an established firm than for a newcomer. (Or offering one more type of soda.)

Economies of Experience

- The economies of experience are described by the **experience curve** which shows a relationship between average variable cost today and $\overbrace{\textit{cumulative}\text{ production}}^{Q_{t-1} + Q_{t-2} + Q_{t-3} + \cdots}$ volume.
- This results in…
  - Greater Labor Productivity
  - Fewer Defects
  - Higher Material Yield

But, how does accumulated experience affect costs?

- Let's look at AVC…

$$AVC(N) = A \cdot \frac{1}{N^B} \text{ where } B \epsilon (0,1)$$

where N is the firm's cumulative production

Or alternatively,

$$AVC(N) = A \times N^B \text{ where } A>0 \text{ and } B \in [-1, 0]$$

- where parameter A is the AVC of the first unit

$$N = 1 \rightarrow AVC = A \cdot 1^B = A$$

- and B represents the "experience elasticity"

Why does B represent the experience elasticity?

Experience Elasticity, *B*

Experience elasticity measures what is the percentage change in the firm's average variable costs, AVC, when the accumulated output, N, increases by 1%.

That is,

$$\frac{\%\Delta AVC(N)}{\%\Delta N} = \frac{\frac{\Delta AVC(N)}{AVC(N)}}{\frac{\Delta N}{N}} = \frac{\Delta AVC(N)}{\Delta N} \frac{N}{AVC(N)}$$

We can write the above expression using derivatives (marginal increase in N) rather than discrete increments, Δ, as follows

$$\frac{dAVC(N)}{dN} \frac{N}{AVC(N)}$$

Let us now apply it to the AVC expression in the previous slide:

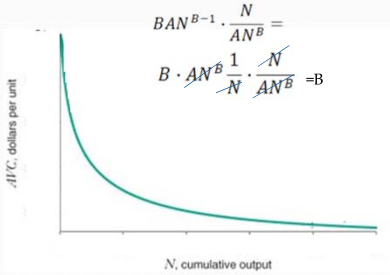
$$AVC(N) = A \times N^B \text{ where } A>0 \text{ and } B \in [-1, 0]$$

Experience Elasticity, B

$$\frac{\%\Delta AVC(N)}{\%\Delta N} = \frac{dAVC(N)}{dN}\frac{N}{AVC(N)} \quad \textit{and since } AVC(N) = A \cdot N^B \ldots$$

$$BAN^{B-1} \cdot \frac{N}{AN^B} =$$

$$B \cdot \cancel{AN^B}\frac{1}{\cancel{N}} \cdot \frac{\cancel{N}}{\cancel{AN^B}} = \text{B}$$



Hence, a 1% change in N produces a B% reduction in AVC

Slope of Experience Curve

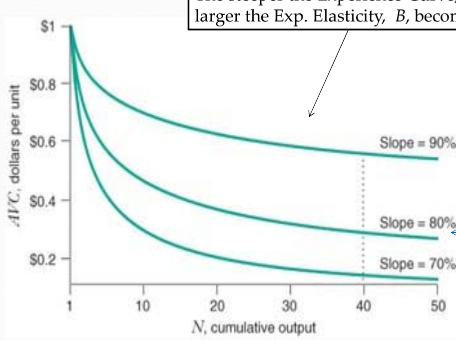
- How much does the average variable cost go down (as a percentage) when cumulative output, N, doubles?

$$\text{Slope of experience curve} = \frac{AVC(2N)}{AVC(N)} = \frac{A(2N)^B}{AN^B} = \frac{2^B N^B}{N^B} = 2^B$$

- Thus, we see a relationship between…
  - The slope of the experience curve (2^B), and
  - Experience Elasticity (B)

Example…

The steeper the Experience Curve, the larger the Exp. Elasticity, B, becomes.



$2^B = .9$ Then B = -.15

$2^B = .8$ Then B = -.32

$2^B = .7$ Then B = -.51

Note: Recall that, in order to solve for 2^B=9, we need to take logs on both sides, i.e., B*log2=log9, yielding B=log9/log2.

Application 8.7 – Experience curves in emissions control

- Gas desulphurization (Reduction of SO_2).
- Catalytic reduction systems (Reduction of NO_x).

Significant Eco. of experience (About 76% reduction of AVC since 1983)

→ If Eco. of experience are ignored in public policy, cost estimates of reducing greenhouse gases may be overstated.

- ***What is the relationship between economies of experience and economies of scale?***

$\uparrow N = \downarrow AVC(\Delta Experience)$

Typical for the production of new products, since after some years the production process gets more efficient.

$\uparrow Q \rightarrow \downarrow AVC(\Delta\, Scale)$

Typical for mature industries

- ***Real World Examples…***
  - Econ of Scale, no econ. of experience: mature industries, e.g., cement, aluminum can manufacturing, etc.
  - Econ of experience, no econ. of scale: handmade products, e.g., handmade watches.

Estimating Cost Functions

1) Constant Elasticity Cost Function

$$TC = aQ^{b} w^{c} r^{d} \quad \text{where} \quad a, b, c, d > 0$$

b is referred to as output elasticity, from total cost. Let's prove it:

$$\varepsilon_{TC,Q} = \frac{\partial TC}{\partial Q} \cdot \frac{Q}{TC} = baQ^{b-1} w^{c} r^{d} \cdot \frac{Q}{TC}$$

$$= b\,(aQ^{b} w^{c} r^{d}) \cdot \frac{1}{Q} \cdot \frac{Q}{TC} =$$

$$= b \cdot TC \cdot \frac{1}{Q} \cdot \frac{Q}{TC} = b$$

c is the total cost elasticity with respect to labor price, *w*.

$$\varepsilon_{TC,w} = \frac{\partial TC}{\partial w} \cdot \frac{w}{TC} = \ldots = c$$

Same step as above

$$c \cdot aQ^{b} w^{c-1} r^{d} \cdot \frac{w}{TC} =$$

$$\underbrace{aQ^{b} w^{c} r^{d}}_{TC} \cdot c \cdot w^{c-1} \cdot \frac{w}{TC} = c$$

d is the TC elasticity with respect to the price of Capital, *r*

$$\varepsilon_{TC,r} = \frac{\partial TC}{\partial r} \cdot \frac{r}{TC} = \ldots = d$$

For an increase in *w* and *r*, the change that it produces on *TC* is

$$aQ^{b}(\lambda w)^{c}(\lambda r)^{d} = \lambda^{c+d} \cdot aQ^{b} w^{c} r^{d} = \lambda^{c+d} \cdot TC$$

Therefore:

1. For the change in TC to be proportional we need $c+d=1$,
2. For the change in TC to be less-than-proportional we need that $c+d<1$.
3. For the change in TC to be more-than-proportional we need that $c+d>1$.

Applying logs in the above total cost curve $TC = aQ^b w^c r^d$

$$\log TC = \log a + \underset{\varepsilon_{TC,Q}}{\underset{\uparrow}{b}} \log Q + \underset{\varepsilon_{TC,w}}{\underset{\uparrow}{c}} \log w + \underset{\varepsilon_{TC,r}}{\underset{\uparrow}{d}} \log r$$

- When running a regression, we find the estimates for these parameters a,b,c,d.
  - We can, thus, interpret them as elasticities.

- Disadvantage of this particular TC function:

  It doesn't allow for economies/diseconomies of scale

  →A given 1% increase in Q produces the same $\Delta b\%$ in TC

- The following TC function allows for economies and diseconomies of scale…

2) **Translog Cost Function**

$$\begin{aligned}\log TC = {} & b_0 + b_1 \log Q + b_2 \log w + b_3 \log r + \\ & + b_4 (\log Q)^2 + b_5 (\log w)^2 + b_6 (\log r)^2 + \\ & + b_7 (\log w)(\log r) + b_8 (\log w)(\log Q) + \\ & + b_9 (\log r)(\log Q)\end{aligned}$$

- It is capable of approximating the TC originating from almost any production function.
- If $b_4 = b_5 = b_6 = b_7 = b_8 = b_9 = 0$ then this function becomes the *constant elasticity cost function* described above
- It allows for economies and diseconomies of scale